PROSPECTIVE MATHEMATICS TEACHERS' VIEWS OF THEIR LEARNING IN A LESSON STUDY

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We aim to identify what prospective mathematics teachers indicated that they learned regarding mathematics and mathematics teaching as they participated in a lesson study. The theoretical framework is supported by models about teachers' knowledge and prospective teachers learning processes. The methodology is qualitative and interpretative, with data collected by interviews and documents made by participants. The participants indicated to understand the importance of different representations in mathematics. They also indicated to appreciate the value of the detailed planning of the lesson, the careful selection of tasks, the analysis of possible student strategies and difficulties, and the preparation of classroom communication.

Keywords: Lesson study; Knowledge of mathematics; Knowledge of mathematics teaching; Prospective teachers

Perspectivas de futuros profesores de matemáticas sobre su aprendizaje en un estudio de clase

Nuestro objetivo es identificar lo que los futuros profesores de matemáticas indicaron que aprendieron con respecto a las matemáticas y la enseñanza de las matemáticas al participar en un estudio de clases. El marco teórico está respaldado por modelos sobre el conocimiento de los docentes y los procesos de aprendizaje de los futuros docentes. La metodología es cualitativa e interpretativa, con datos recogidos por entrevistas y documentos producidos por los participantes. Los participantes indicaron comprender la importancia de las diferentes representaciones en matemáticas. También indicaron apreciar el valor de la planificación detallada de la lección, la cuidadosa selección de tareas, el análisis de posibles estrategias y dificultades de los estudiantes, y la preparación de la comunicación en el aula.

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Términos clave: Estudio de clases; Conocimiento de matemáticas; Conocimiento de la enseñanza de las matemáticas; Futuros profesores

Perspectivas de futuros professores de matemática sobre as suas aprendizagens num estudo de aula

O nosso objetivo é identificar o que os futuros professores de Matemática indicaram que aprenderam em relação à Matemática e ao ensino de Matemática ao participarem de um estudo de aula. O referencial teórico é apoiado por modelos sobre o conhecimento dos professores e os processos de aprendizagem dos futuros professores. A metodologia é qualitativa e interpretativa, com dados recolhidos por meio de entrevistas e documentos realizados pelos participantes. Os participantes indicaram compreender a importância das diferentes representações em Matemática. Também indicaram apreciar o valor do planeamento detalhado da aula, a seleção cuidadosa de tarefas, a análise de possíveis estratégias e dificuldades dos alunos e a preparação da comunicação em sala de aula.

Palavras-chave: Estudo de aula; Conhecimento de matemática; Conhecimento de ensino de matemática; Futuros professores

Quality preparation of prospective teachers is important to support quality mathematics teaching. Prospective teachers, among other aspects, need preparation in mathematics and in mathematics teaching. With the rapid changes occurring in the current society and the need to respond to new educational challenges, this preparation must consider not only the existing school reality, but also empower the prospective teacher to become an active member of a changing educational system. As Ponte and Chapman (2008) indicate, this is not an easy task:

> *[I]t is very complex to provide preservice teachers with effective competence for teaching according to the reform orientations. In a sense, what is asked of preservice teacher education is an impossible task. In a short time, it must prepare a young, perhaps rather immature person to assume a highly complex professional function. We should not overlook the fact that teaching mathematics involves conducting mathematical activity with large groups of students [...] who often have little interest in mathematics, have serious affective and social needs, and come from increasingly mixed cultural settings. Further, we ask new teachers to engage in practices that are compatible with innovative curriculum orientations, but which generally are not the established school approach (pp. 255–256).*

Therefore, it is necessary to explore forms of optimizing the available resources to guarantee the best possible preparation to prospective teachers. Among the most common critiques to initial teacher education is the separation between theory and practice and the fact that teacher education tends to unfold in unconnected activities that the prospective teacher finds difficult to integrate as a coherent whole (Lampert & Ball, 1998). Lesson study, carefully framed considering the conditions of teacher education programs, is a promising working process for initial teacher education, allowing overcoming these limitations. However, it is necessary to understand what learning aims must be set for prospective teachers and to consider the consequences of different possible working processes. Currently, research carried out in this field yields more questions than answers (Larssen et al., 2018; Ponte, 2017). An important source of information may be the perspectives of prospective teachers when they participate in lesson study processes. So, the research question of this study is: what prospective mathematics teachers that participated in a lesson study, from their own point of view, learned regarding mathematics and mathematics teaching?

Lesson Study

Lesson study is a professional development process highly practiced in Japan that was disseminated to the Western world in the book "The teaching gap" (Stigler & Hiebert, 1999) and achieved great interest in recent years in many countries. In lesson study, a group of teachers begins by identifying a students' learning problem and, studying teaching materials, research articles and other sources, seek to understand how to overcome that problem. Then, they prepare a lesson to address that problem, designing a detailed lesson plan in which they indicate the tasks to present to the students, the possible strategies that the students may follow, and the actions that the teacher may use in response. A teacher of the group teaches the lesson, called research lesson, and the other teachers observe the lesson, paying special attention to students' participation. Finally, in a post lesson reflection, the teachers discuss the evidence that they gathered of students' learning and consider possible modifications to make in the lesson regarding the tasks, the flow of the lesson and the activity of the teacher (Fujii, 2016; Lewis et al., 2019). Therefore, lesson study may be viewed as a small investigation of the participant teachers of their own practice, as they frame a research problem, carry out a review of literature and other materials to address that problem, design and make an experiment, collect data, and analyze that data (Ponte et al., 2016). Three important features of lesson study are its focus on student learning and its reflexive and collaborative nature (Quaresma & Ponte, 2021). Japanese researchers underline the close relationship of lesson study and the mathematics curriculum approach that they characterize as "structured problem solving" (Fujji, 2018). In this approach, that may be also named as "exploratory teaching" (Ponte & Quaresma,

2016), student learning is not based in listening to lectures and practicing exercises but in working from tasks and discussing different strategies to solve them.

Lesson study is, usually, developed as a professional development process for practicing teachers. However, teacher educators have also considered its possible use in initial teacher education, supporting the development of prospective teachers' knowledge in connection with the context of teaching practice. Previous research about the development of knowledge in lesson studies of secondary school mathematics teachers signals several elements about what prospective teachers may learn and what difficulties they may experience. For example, Bjuland and Mosvold (2015) indicate that the possible benefits of lesson study may be lost if prospective teachers do not formulate a research question for the research lesson, do not focus on observing students' learning, or do not organize the lesson for making students' learning visible. Fernández (2005) shows that prospective secondary teachers may experience difficulties in their mathematical knowledge in specific topics, which may be addressed during the lesson study activities. Regarding didactical aspects, previous studies indicate that prospective secondary teachers may learn about lesson planning (Burroughs & Luebeck, 2010) as well as about students' strategies and learning difficulties (Martins et al., 2021). In our review of literature, we did not identify studies addressing the perspectives of prospective secondary school mathematics teachers regarding their participation in lesson study.

Prospective Teacher Knowledge

One of the main aspects of the preparation of prospective mathematics teachers concerns the development of their knowledge about mathematics and mathematics teaching. Current discussions about these fields of knowledge are still framed in a decisive way by the seminal work of Shulman (1986, 1987) who called attention for a kind of knowledge that he considered neglected in initial teacher education—pedagogical content knowledge. This knowledge, in his view, establishes a fundamental articulation between pedagogy and content. Shulman (1987) refers that it is also necessary to consider other domains of knowledge, such as knowledge of the curriculum, of learners, of educational contexts, and general pedagogical knowledge. However, with time, *content knowledge* and *pedagogical content knowledge* become seen as the fundamental nucleus of the knowledge for the prospective teacher and has been object of many studies (Ponte & Chapman, 2008; Strutchens et al., 2017). Seeking to bring more precision to the content of these two knowledge domains, Ball et al. (2008) proposed a model that introduces several subdomains. In content knowledge, these authors underline the importance of 'specialized content knowledge' and, in pedagogical content knowledge, they highlight 'knowledge of content and students'. In fact, the need that teachers understand students' thinking and learning processes was already a major idea of

the research program Cognitively Guided Instruction (Carpenter & Fennema, 1992), and currently constitutes a key element of the preparation of prospective teachers in many initial teacher education programs.

Another perspective about teachers' knowledge is given by Rowland et al. (2005). The work of these authors also has Shulman (1986) as a starting point, but instead of defining "domains" or "subdomains" they consider "dimensions". They establish four fundamental dimensions that they argue enable the description and analysis of the teacher's work: foundation, transformation, connection, and contingency. This model strives to capture the dynamic of the teacher's work, seeking to consider what happens as the teacher prepares a lesson and leads the activity in the classroom.

Following the European tradition of defining didactics as the study of teaching and learning in different curriculum subjects, Ponte (2012) also underlines the connection between teacher knowledge and teaching practice. This author highlights the central role of the teacher's knowledge of the teaching process and that includes several key concepts to develop a practice aligned with the curriculum frameworks, particularly the notions of task, representation or solving strategy. He also highlights the role of classroom communication, both in supporting students' autonomous work and in leading whole-class discussions.

Regarding the processes of development of teachers' knowledge, Ball and Cohen (1999) consider practice as a fundamental element. The authors do not claim that practice, by itself, provides all required knowledge, but argue that the reference to situations of practice, the work with situations of practice and the reflection about experiences of professional practice are essential elements in the process of development of teachers and prospective teachers, if properly analyzed and framed according to the curriculum and relevant educational notions.

Lesson study allows establishing a relation among these theoretical perspectives. On one hand, it has a strong connection with practice, developing around the preparation, undertaking, and reflection of a lesson. On the other hand, lesson study requires the mobilization of knowledge of the content, regarding concepts, procedures, solving strategies and representations and of didactics knowledge, in key issues such as lesson planning, selection of tasks, and analysis of students thinking processes and of classroom communication. Particularly important is the role of multiple representations, notably symbolic, graphical, and verbal (Lesh et al., 1987; NCTM, 2014). These aspects will be central in the analysis in this paper.

Research Methodology

The methodology of this study is qualitative and interpretative (Erickson, 1986). The participants are nine prospective mathematics teachers, who were attending the second semester of the first year of their master's of teaching program in a

Portuguese university[1]. These prospective teachers constituted the whole cohort attending this program in this academic year. All prospective teachers were asked for their participation in the research. They had a bachelor's degree in mathematics (three years) or on a related field with strong mathematics preparation. Three of the nine prospective teachers had teaching experience and were at the time teaching mathematics or other subjects under provisional contracts. They already had a course on mathematics didactics at the first semester of the program and they were attending a second didactics course in this semester. The first didactics course dealt with issues of mathematics curriculum and classroom teaching, including tasks and classroom communication and the second didactics course considered learning resources, including technology, student evaluation, and lesson planning. The activity described in this article took place as a regular activity of a course on initiation to professional practice that included work at the university and observation in school contexts. The course instructors were the two first authors.

Data was collected by a semi-structured interview carried out at the end of the semester (FI) and collection of the prospective teachers' final individual written works (IW). The interviews were made by the authors of this article and were fully transcribed. We coded the data according to a system of categories that in part was based in the conceptual framework and in part emerged from the data. For the category knowledge of mathematics, the subcategories were: (i) Awareness of own difficulties in mathematics and (ii) Solving strategies and representations. These subcategories emerged from the data. For the category knowledge of mathematics teaching, the subcategories were: (i) Lesson planning; (ii) Selecting tasks to propose to the students; (iii) Anticipating the strategies and difficulties of students; and (iv) Conducting classroom communication. These subcategories were based in the conceptual framework[2]. The first author coded the data, and the other two authors verified it. The three authors discussed the cases of disagreement in depth to reach a consensus. This research was approved by the Ethics Committee of the Instituto de Educação da Universidade de Lisboa, statement of 24 April 2020.

The Lesson Study

This lesson study was carried out with a class with nine prospective mathematics teachers of a master's of teaching program in a Portuguese university. It took place in the second semester, within the frame of a field experience course that usually includes school visits and reflection about those visits. However, the teacher education program does not seek that prospective teachers teach lessons at this stage, which only happens in the third semester. The course had two co-teaching

1 All referred by fictitious names.

[2] The analysis of data considered still other categories, drawn from the literature or emerging from the data. In this article, we discuss the categories in which the results were, in our view, stronger and more interesting.

instructors (the first and second authors). After each class session, the instructors met to debrief and to plan in detail the next session.

The lesson study developed in four strands, addressing different aspects of teacher knowledge. The first strand included a work focused on mathematics, solving mathematical tasks, and reflecting on solving strategies and representations. The second strand addressed school students' strategies and difficulties on the topic. The third strand concerned the detailed planning of two lessons, considering the curriculum framework. Finally, the four strands included the observation and reflection on the lessons.

The lesson study had nine sessions (2 hours each), and two additional research lessons that were carried out in the classes of an experienced teacher. This teacher collaborated all along the process. The analysis of the school curriculum planning and the university calendar circumscribed the possible topics to address. The selection was made in the second session, with the participation of the schoolteacher, choosing the topic "Solving equations and inequalities with absolute values" in grade 10, in which the students tend to experience many difficulties. As the lessons in this school are 100 minutes long (with a small break in the middle), the prospective teachers worked in two groups formed at their own initiative, one with four and the other with five participants. Each group prepared a 50 minutes lesson in collaboration with the teacher and the instructors of the course. The group analyzed several curriculum documents to understand the way the curriculum dealt with the topic. It also analyzed didactics questions, with special attention to the relation of tasks and sequence of tasks with learning goals and the identification of different kinds of classroom activity (launching, autonomous work, whole-class discussion) and ways to support students in each one of them. The discussion of these issues was based in professional articles previously read by the prospective teachers.

At the end of the second session, we asked the prospective teachers to identify possible tasks regarding the lesson study topic (work to carry out in pairs). At the third session, the prospective teachers solved some tasks, selected so that they would represent a variety of situations and difficulty levels. The aim was to address the main mathematical aspects of the topic and underline possible students' strategies and difficulties. At the end of the session, prospective teachers were assigned the reading of several pages of a research article about students' difficulties in this topic (Almog & Ilany, 2012).

In the following sessions, the group discussed the assigned pages of the article, analyzed how school textbooks presented the topic and defined the goals for the two research lessons. The first lesson would address the solution of tasks involving equations and inequalities with absolute values only in the left member of the expression and in the second lesson tasks involving more complex equations and inequalities, with absolute values in both members. We must note that at the time of teaching the lessons, the students of this class had already learned the absolute value function but had not learned to solve equations and inequalities with absolute

values. The prospective teachers worked in defining the tasks for the lessons and in constructing the first version of the lesson plans. They presented their proposals in class to improve them. The classroom teacher had an active participation in all sessions of the planning of the research lessons.

The seventh session addressed the preparation of the observation. The group considered general issues regarding observation, considering that the prospective teachers will have to make observations in other courses of the teacher education program and with other aims. Also discussed were observation methods and processes, with special attention to observation foci and observation records. This was closed with the preparation of the observation of the lesson ─what would be observed and how.

Immediately after the two research lessons there was a first reflection (about 30 minutes for each lesson). Finally, there was a last session with a deeper reflection about the research lessons and a general evaluation of all the work carried out in the lesson study.

In addition to the reading and the tasks proposed along the sessions, the prospective teachers completed two main written works. The first, individual, was delivered before the day of the research lesson, and should describe the main aspects of the planning of the research lesson and include a written reflection about what the prospective teacher had learned so far. The second, in-group (corresponding to the two groups that planed the lessons), addressed all work carried out, with special focus in what was observed in the research lesson, in the light of the lesson plan.

RESULTS

Knowledge of mathematics

Awareness of own difficulties in mathematics

The topic of the lesson study was solving equations and inequalities with absolute values. We asked prospective teachers to propose tasks concerning this topic and some of these tasks (representing several levels of complexity) were collectively solved at one of the first sessions of the lesson study. Most prospective teachers were surprised as they faced difficulties in solving the tasks. The difficulties were varied, but the most notable concerned the use of logical connectors ‘and’ and ‘or’ and solving questions involving second degree expressions in both sides of an equation or inequality:

> *Carla, FI:* First, I did not expect that I would have so many difficulties. I also saw difficulties in my colleagues, that I did not expect them to have [...] Yes, for example that question of [brackets] to represent ‘and’… I think that I never had thought about that before.

Sofia, FI: Concerning the question of equations and inequalities with absolute values, the way we detailed that content, the associated difficulties and everything else, it allowed to see the difficulties that I did not know that I had and finally I had in that content [...] Yes, we had many questions.

It is possible that the fact that they had studied this topic in their academic journey led the prospective teachers to think that they were prepared to solve the tasks usually proposed to students. In addition, their university mathematics preparation, during which they overcome many obstacles, very likely contributed to reinforce the idea that they were prepared to teach this topic and all the other topics with no major problem. Therefore, they were not expecting to face the difficulties that they noticed.

Several prospective teachers recognized that the work carried out in this and in the following sessions helped them to overcome the difficulties that they felt:

Carla, FI: I learned many things. In fact, we had more difficulty in that [using connectors]. It was where I learned more, no doubt.

Sofia, IW: Now I know that I must be prepared, as prospective teacher, to analyze the topics to teach, so that it allows the students to remember what they learned before.

In this way, the prospective teachers valued the work carried out in exploring mathematical tasks, assuming that is a practice to follow in their work as teachers. They seemed to have understood that it is not enough to have studied before a subject to recall it with rigor when they teach it. They signaled the need to consider the previous knowledge that they needed to solve each task and the importance that they feel confident in their domain of the mathematical topics, for their interaction with students.

Solving strategies and representations

Several prospective teachers indicated that during the lesson study they learned about the topic. For example, a prospective teacher referred that before she only used to solve processes for inequalities with absolute values based in the algebraic representation and in the use of the definition and basic properties of absolute values and recognized that the work done led her to learn new solving processes. She also recognized that it is important to know that, in solving a mathematical task, often several strategies may be followed:

Sofia, IW: I always had the perception that the analytical solution has a higher value, and it is that one that we are "supposed" to teach to our students, since it draws on definitions and properties learned in class. I also learned new ways of solving inequalities with absolute values that never occurred to me and that, in fact, it is important to have several perspectives how one may solve a given task.

Another prospective teacher indicated that she improved her knowledge of the topic, notably methods for solving equations and inequalities with absolute values, that she did not know and that she found surprising. In addition, she also referred to her learning about the characteristics of these methods, recognizing the underlying representations and strategies as well as the previous knowledge necessary:

Carla, IW: When we studied the exercises that we brought we also identified some possible reasoning for solving tasks, some of which were surprising for me. Sometimes, small steps within a solution, other times another solution that I did not identify, but all of them allowed to improve my knowledge about this topic [...] To qualify the solution methods used and the knowledge necessary for each step was crucial for a wider understanding of the topic. For example, to identify the cases when the definition of absolute value is used or its properties, and the use of the properties of equations and inequalities. It is also important to know to classify a solution as graphical or algebraic, among others.

Several prospective teachers were used to solve the proposed tasks using only the algebraic representations. The use of graphical representations was a surprise for them:

Ronaldo, FI: Yes, I had thought that they had several solutions, but I never thought that the easiest, or simpler, to learn would be the graphical. I never thought that [...] I was rather surprised because I always thought [...] that the algebraic representation would be easier for students to learn but [...] I recognize that I was wrong and that in some tasks the graphical representation helps them to understand [...] easily the situations. I recognize that, in fact, it went against [what I expected].

Sofia, FI: At school I never was encouraged to use it [the graphical representation] because I thought that the solution that teachers prefer is algebraic, therefore I always used the algebraic [...] Now I consider that the graphical [representation] is more important.

The work carried out led the prospective teachers to recognize the importance of using the graphical representation in this kind of task, allowing the students a better understanding of the situations.

A prospective teacher with some teaching experience indicated that he did not have any mathematical difficulties, but, even so, he recognized that he learned something, notably regarding the way to approach the topic:

Nelson, FI: Even on a topic that I thought that I knew, I learned new ways of approaching it that enrich the way I can later present it. That is, to know a certain topic does not mean that it can't be tackled in other ways and that was what I obviously learned in this part of the journey, not much specific things about the topic [...]. Often, we have the idea that we know, but we don't know everything and, therefore, searching other

> forms of knowing, even in mathematics that seems to be something that we know or don't know, it was that I found interesting to learn.

In conclusion, several prospective teachers presented interesting reflections about what they learned regarding the strategies for solving tasks, recognizing that, for a given task, a variety of solutions may exist. It should be noted that, in the session in which mathematical tasks were solved, it was immediately evident that prospective teachers did not use the graphical representation to solve tasks about equations and inequalities with absolute values, because they did not know it, they did not value it, or because they felt it would not be accepted as a proper solution. The need to propose tasks that may involve a variety of representations and the role of the graphical representation may assume in solving these tasks was a constant aspect of the work carried out and constituted a significant learning for the prospective teachers.

Knowledge of mathematics teaching

Lesson planning

All prospective teachers, albeit emphasizing different aspects, described significant learning concerning the elaboration of lesson plans. Some of them even indicated that it was the most important learning that they made in the course.

> *Nelson, FI:* I learned… It was even where I learned more [...] The detail that we learned was much greater than I had ever used. Regarding this, I learned a methodology and a systematization that may be useful to me.

> *Paulo, FI:* I never had seen a plan with this length and with all this detail and from the moment in which we made the plan, and we enacted the plan, that is, we made a full lesson study, we were able to understand all steps of the preparation of a lesson, what was great.

In their reflections, the prospective teachers especially referred the detailed nature of the planning carried out and underlined the care that is necessary in selecting tasks. In addition, they highlighted the fact that the lesson planned was taught, allowing a global understanding of the aspects involved in planning a lesson.

An exploratory lesson requires a sort of planning different from a lecture. Several prospective teachers showed an understanding of this issue, indicating that they would seek to include exploratory lessons in their future teaching practice:

> *Sofia, FI:* The importance of exploratory lessons, mainly the lesson that we went to see, the way the lesson unfolded and how it developed, that, in my opinion, was a success for students. I thought it [...] would be something difficult to implement [...]. It seemed something very complicated that the students would get there by themselves, without our help. It was something I thought [to be impossible] … But finally, it is quite possible!

Paulo, FI: It has advantages [...] students learn in a more autonomous way and teaching becomes easier for students... For the teacher, it may become a little more complicated, but, in a general way, for the student it is very beneficial. Learning is much easier [...]. It is much more difficult [to prepare] an exploratory lesson... Lecturing is to expose the subject. The teacher comes there and presents the subject [...] In a new subject, it may be important to begin with an exploratory process so that the student may understand.

Prospective teachers recognized that they learned to do lesson plans in a way quite different of what they knew and appreciated the importance of their detailed nature. In other courses they had contact with the exploratory lesson model, but they indicated that they had visited several schools and never had seen such kind of lesson. For them, to do exploratory lessons, seemed to be an interesting idea, but difficult to carry out in practice. They recognized that the two lessons that they planned assumed an exploratory nature and that those lessons unfolded according to the plans. In this way, they valued this experience that provided them the direct observation of exploratory lessons. They recognized that this kind of lesson may lead to important students' learning and that, after all, it is not very difficult to put into practice, albeit it requires more work in its preparation.

Selecting tasks to propose to the students

Another aspect that was also much referred by prospective teachers also concern the careful selection of tasks to propose to students in the classroom. They indicated that the choice of tasks must consider the learning goals that the students should aim, and be diversified in terms of representations and strategies that may be used:

Nelson, IW: The kind of tasks to propose in the classroom must be selected according to our aim and the purpose that we define for the task. We must decide if the task should be an exercise, problem, exploration, or investigation. We also must have into account the more relevant aims that tasks to propose to students must have, as, for example, assess, develop mathematical language, construct, and understand concepts, create connections, procedures, representations, etc.

Sónia, IW: Tasks are utmost importance, and among other factors, they must pay clear attention the goals to achieve. They must be mathematically challenging, allow the diversification of mathematical representations, of reasoning, and have an open structure. However, they must not be so complex that induce students a feeling that they cannot do mathematical work.

In this way, Nelson listed several kinds of task that the teacher may propose and indicates possible aims that the teacher may have in selecting them for the students.

Sónia referred that tasks must be challenging, but despite that, be within the range of students' capacity.

In addition, Carla referred the need to establish connections between what students had learned and what they will learn anew and reflected on the importance of working with tasks with different levels of demand, if all have their role:

> *Carla, IW:* For me, one of the most complex tasks in this journey was the selection of tasks to propose in the classroom. As I referred, I consider that to understand that it is not easy to understand the difficulties of students and that may lead to choose less appropriate of exercises. It is necessary to have into account the themes already worked with students in order to consider the best way to make a bridge between what they studied and what is new. Therefore, the kind of tasks may be key to guide the lesson. Tasks that may create more difficulties in students may also be those that promote more communication among students. On the other hand, routine exercises must be proposed as homework.

In this way, Carla shows how to understand how the choice of appropriate tasks to propose to students is an activity that requires much attention from the teacher.

Another prospective teacher, Sofia referred that she understood that, instead of selecting many tasks, it is better to select a smaller number. This allows that their solutions are done in a detailed way in order to support students' learning:

> *Sofia, IW:* The tasks to propose in the classroom must be very well thought and very well selected in order to address all aspects thought to the lesson with the minimum number of tasks [...] "Less is more" regarding the number of tasks to do. The more tasks to propose, less detailed and less discussed will be the solution and higher the risk that more students do not grasp all that the task had to teach [...]

It seems like this prospective teacher internalized the need for attention to details that informed all lesson study, regarding the planning of the lesson and the possible students' solutions and difficulties.

In summary, several prospective teachers explicitly referred the problems involved in selecting tasks to propose to students. They underlined that this selection must be done in a careful way considering the learning aims to be achieved by their students. They referred the need to propose diversified tasks, including challenging tasks, but at the same time, accessible to students. In this way, the issues to consider when selecting tasks, seems to be a significant learning achieved in this course.

Anticipating the strategies and difficulties of students

Another important learning highlighted by prospective teachers is the need to pay attention to students' strategies and to anticipate their difficulties. They described, for example, that while making a lesson plan, it is necessary to know what reasoning and difficulties the students may have:

Sofia, IW: Students' reasoning and difficulties is one of the more important topics of the lesson plan, in the sense that if we anticipate as much as possible students' reasoning, as well as possible difficulties, we may prepare the way how we overcome such difficulties and the lesson will unfold much better.

Nelson, IW: In planning a lesson, students' reasoning and difficulties are key aspects in previous knowledge that we must know [...] Previous knowledge of the way they usually reason, their more common characteristics and difficulties, the kind of work that has been done on the subjects related to this topic is fundamental to plan a lesson that the students may enjoy and from which they all may benefit [...]

Often, the analysis of tasks to propose to students is done in a superficial way, presupposing that as the lesson unfolds, the teacher will know to respond to eventual difficulties that the students may experience. However, many unforeseen difficulties may emerge, with incorrect assessments of the reasoning processes necessary for solving them. Several prospective teachers referred the importance of previous careful solving of tasks in order to identify possible students' difficulties:

Ronaldo, FI: As I already teach [...]. I made, we need to teach this, follow this sequence, but I didn't foresee the students' difficulties and how to respond to them... For me, that part was what would arise on the spot. A little bit at random [...] In this course, I learned that in fact we can foresee these things and must foresee them [...] It was one of the first thing that I learned.

Sónia, IW: Regarding students' reasoning and difficulties, it was the domain in which I learned most. To carry out some tasks on the theme, considering the students' perspective, allowed me to foresee some features that, on the first sight, seem irrelevant, but that in fact represent a barrier to students' progress in solving [the task] and, therefore, are a barrier in their relations with mathematics. I understood the importance of anticipating those difficulties as the students solve the task.

Ronaldo, who already had teaching experience, pointed that identifying possible students' difficulties is an aspect that he would change in his practice. Sónia even referred that this was the most significant learning that she made in this course.

In addition, Tânia referred to the way she may know students' reasoning and difficulties and how she may support them:

Tânia, IW: It is also important that the teacher formulates questions that allows access to students' mathematical reasoning, in particular to the understanding that students have of important concepts related to the lesson aim [...]. A way [to support students in overcoming difficulties]

> consists of formulating questions to "unblock" students that at some point of solving the task, can't proceed any longer.

In summary, the importance of the teacher identifying the possible students' strategies and difficulties was a strong feature of the work carried out. The prospective teachers seemed to value this, as they make explicit reference to this issue, indicating a variety of aspects. They indicated the complexity of this identification; they pointed the need of this work for elaborating the lesson plan; and refer the crucial role of solving tasks related to the topic. As it was done in the lesson study, they referred, that they may search for research studies, they pointed the relation of the difficulties with previous knowledge, and they indicated the need to know how to overcome such difficulties. This point seems to be a rather consistent learning of this course.

Conducting classroom communication

In planning the research lesson, much attention was given to the way the teacher would conduct classroom communication. The prospective teachers recognized that the way communication is conducted is essential to students' learning:

> *Carla, IW:* Classroom communication is essential for its unfolding. I consider that I developed knowledge in this matter, since, at least in these lessons that were prepared, I managed to understand the aim of each communication element.

> *Nelson, IW:* The way to conduct classroom communication will be much better as one foresees, in planning the lesson, students' responses/reactions regarding the proposed tasks, as well as the possible difficulties that they will experience. So, we may plan the responses and the aspects to consider in conducting the lesson and in the communication actions to use and to promote.

Conducting communication during whole-class discussions puts a serious challenge to the teacher. This classroom moment gathered large attention during the planning of the research lessons. Several prospective teachers referred to this issue:

> *Carla, IW:* It is important that the teacher is sensitive to understanding what the moments are to carry out a discussion, for example. The way to guide the discussion as well as the kind of questions that one puts to students are also important for the discussion to allow the consolidation of knowledge. In the moments of autonomous work, the teacher must be able to support the students, when necessary, in a strategic way. He/she must not be much explicit showing the response to the students, [but] answering the question so that the students think about them.

> *Tânia, FI:* I get excited, mainly the discussion among students and among teacher and groups [...] Because sometimes I think that the students do not question the things that they learn. I mean, they, when they discuss

> things with each other, they already have a critical eye regarding what they are learning. Sometimes, I think that there are many students, and the teacher says one thing and they accept, they do not even think in what they are saying.

In their initial proposals for lesson plans, the prospective teachers indicated that a synthesis should be done at the end of each moment of whole-class discussion. However, these proposals just signaled that the synthesis should be done, with no further indications. The issue of the aspects to include in the syntheses received attention during the planning phase. This issue was highlighted by two prospective teachers:

Nelson, FI: And we understand that the synthesis is one of the most important things in the lesson plan. All the work ends up being systematized in that final phase. That was a thing that we felt as we made this journey.

Sónia, FI: We also understood that finally the syntheses are extremely important and must be thought of.

The need to take into attention the way to conduct communication was an aspect addressed in this course, with special attention to carrying out whole-class discussions. The prospective teachers showed to develop their awareness regarding this aspect of the work of the teacher.

DISCUSSION

When they carried out this lesson study, the prospective teachers had already a strong mathematical preparation. Therefore, it was with surprise that some of them experienced difficulties in solving tasks about equations and inequalities with absolute values. Even those that solved all tasks (as in Fernández, 2005), they learned new solving strategies. However, more than learning new specific procedures of solving these tasks, it is noteworthy their learning about the role of different representations in mathematics, notably, in this case, the power of the graphical representation. Several prospective teachers were used to approaching these tasks using only the algebraic representation and considered that it was the single rigorous way to solve them. With the work carried out in the lesson study, they developed an understanding of the value of articulating several representations, taking advantage of the possibility of interconnecting them. Therefore, more than learning new concepts or procedures, the participants developed their views regarding mathematics solving processes.

The prospective teachers had already learned about mathematics teaching in the previous semester in the course. However, they referred that they still learned several aspects with the work carried out in the lesson study. They mostly highlighted the way to plan an exploratory lesson, the key role of tasks in such planning and the need that these tasks are appropriate to the aims set, that they are challenging but accessible to students, and allow the use of a variety of

representations and strategies. They also learned the importance of anticipating students' strategies and difficulties, recognizing that this requires careful preparation that will provide the teacher more confidence while teaching the lesson. They also valued the role of classroom communication, especially in preparing and conducting whole-class discussions.

They achieved such learning in the frame of a practical activity of planning and observing two lessons, which provides much more consistency than by undertaking unconnected activities, as it is so common in initial teacher education programs (Lampert & Ball, 1998). However, carrying out this activity is only like opening a door. It allows the prospective teacher a global perception of the planning process and its several elements but does not provide an effective competence. They may only achieve this with new learning experiences and later, in professional practice. In addition, we note that this planning process had its limitations. The bigger of these was the very limited knowledge prospective teachers regarding the students that would work on the tasks. This knowledge was introduced in the lesson study by the schoolteacher whose participation in this part of the work was essential to guarantee the suitability of the tasks. In the future, when they have their own classes, the knowledge of their students will be an important aspect that the prospective teachers will need to know how to handle in their practice. Therefore, what the participants learned about mathematics teaching will need to be deepened and developed in later work.

A fundamental option followed in this lesson study was to carry it out around the preparation of two research lessons that were taught by an experienced teacher. Attending to the moment in which they were in their professional development process, we considered that it was too early for the prospective teachers to teach those lessons. Therefore, during the lesson study, there was a fruitful collaboration involving several actors—prospective teachers, university instructors, and schoolteacher. The lesson study highly valued the work about mathematical concepts and representations, in order to point to prospective teachers, the need to pay attention to the mathematical aspects, alongside with the didactical ones. Finally, special attention was given to key aspects of knowledge about mathematics teaching that are central to an exploratory approach (Ponte & Quaresma, 2016), such as tasks and classroom communication, seeking to show to prospective teachers the viability of putting into practice this kind of teaching.

Conclusion

The results of this study indicate that combining work on mathematics tasks with analysis of sample of students' work (Smith, 2001), along with the detailed planning of lessons and the reflection on the enactment of these lessons are very fruitful ground to develop secondary school mathematics prospective teachers' knowledge, especially because of the collaboration of an experienced

schoolteacher along all the lesson study process. It also shows that, in lesson studies with such organization, these prospective teachers may develop a deeper knowledge of content, becoming more aware of their own limitations in mathematics knowledge as well as regarding crosscutting issues such as solving strategies and representations. Regarding didactics issues, besides lesson planning that was pointed by previous research (Burroughs & Luebeck, 2010; Ponte, 2017), this study shows that secondary school mathematics prospective teachers may experience significant learning regarding the selection of tasks to propose to students, anticipating students' strategies and difficulties, and conducting classroom communication, both during students' autonomous work and during whole-class discussions.

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