

Design of a rehabilitation device for thrombosis: a mathematical modelling activity in the training of engineers

Diseño de un dispositivo de rehabilitación para la trombosis: una actividad de modelización matemática en la formación de ingenieros

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Abstract ∞ The research reported here is framed in the problem of proposing and analysing conducive mathematics teaching for future engineers. It considers that establishing relationships between mathematics and engineering courses is the first step towards training mathematically-competent engineers. In the frame of the Anthropological Theory of the Didactic, one approach to this problem consists of analysing mathematical modelling activities in engineering and transposing it to school. This work presents how a group of engineering students developed a rehabilitation device for thrombosis, relating different types of knowledge: mathematical, engineering, and practice come from different courses and the investigations made for this project.

Keywords ∞ Mathematical models; Training of engineers; Mixed-praxeology; Didactic engineering; Study and research paths

Resumen ∞ La investigación aquí reportada se enmarca en el problema de proponer y analizar una enseñanza de las matemáticas propicia para los futuros ingenieros. Se considera que establecer relaciones entre los cursos de matemáticas y los de ingeniería es el primer paso para formar ingenieros matemáticamente competentes. En el marco de la Teoría Antropológica de lo Didáctico, una aproximación a este problema consiste en analizar actividades de modelización matemática en ingeniería y transponerlas a la enseñanza. En este trabajo se presenta cómo un grupo de estudiantes de ingeniería desarrolló un dispositivo rehabilitador para la trombosis, relacionando diferentes tipos de conocimientos: matemáticos, de ingeniería y prácticos, de diferentes cursos y de investigaciones realizadas para desarrollar este proyecto.

Palabras clave ∞ Modelos matemáticos; Formación de ingenieros; Praxeologías mixtas; Ingeniería didáctica; Recorridos de estudio e investigación

1. INTRODUCTION

The teaching of mathematics has played an essential role in the training of future engineers. Although training models are evolving, as noted by Pollak (1988), Bourguignon (2001), and Graham (2018), they do not seem to do so at the speed that professional practice does. Notably, in some universities, mathematics courses continue to be proposed without necessarily considering their relevance to other courses and professional practice. In effect, the failure of connections that exists between mathematics and other courses has been highlighted in numerous studies (e.g., Castela & Romo, 2011; Faulkner et al., 2020; Gonzalez-Martin, 2018; Harris et al., 2015; Hochmuth, 2020; Rønning, 2021; Tribus, 2005). Tribus (2005) emphasises that in the first semester, mathematics is supposed to represent the everyday discourse that makes it possible to relate such basic subjects as mathematics, chemistry, physics, electronics, acoustics, and mechanics to more specialised courses. Achieving this supposition or goal requires specific work. According to Castela and Romo (2011), the mathematics used in engineering courses can be analysed through the way in which a mathematical technique is validated and used, and the types of tasks performed (mathematical, practical, engineering, etc.). For instance, in the control theory courses analysed, the existence of Laplace transform, or inverse transform, is rarely problematised because validation depends more on their uses. However, these non-explicit questions are treated in the case-by-case analyses of each function (unitary step, ramp, and impulse). Through 'mathematics in use', Faulkner et al. (2020) analysed the concepts and skills learned in calculus and identified the ones used in tasks involved in statics and circuits. Results show that calculus concepts are used in 8% of statics tasks and 20% of circuit tasks, the two kinds of problems that applied the most straightforward calculus skills. In conclusion, statics and circuits courses use very few concepts and skills of calculus. In general, the tasks proposed are mostly solved with algebra. Similarly, Gonzalez-Martin (2018) have shown that the integral (a notion studied in calculus courses) is used through tables and formulas in courses on the strength of materials.

Harris et al. (2015) mention that mathematics is problematic for engineering students. Their findings led them to interpret the pedagogical practice of teaching non-contextualised mathematics as consisting in a lack of transparency regarding the importance of mathematics for engineering. Their research concludes that students' perceptions would improve if engineering programs included problems that are more appropriate to their disciplines, a challenge that demands a specific didactic design methodology (Bartolomé et al., 2019; Siero et al., 2017). Proposing contextualised tasks requires identifying and analysing engineering, or real, tasks and recognising their relevance to be transposed to engineering teaching. For example, over 160,000 cases of venous thromboembolic disease occur in Mexico annually, and some of the people with this disease require specific forms of rehabilitation. One engineering teacher had this condition, which led him to identify that students could create a rehabilitation prototype for ankles affected by deep vein thrombosis. In particular, a prototype similar to those recommended by orthopaedic surgeons for this type of rehabilitation has the movements of flexion,

extension, adduction, abduction, and circumduction. This situation led us to generate two research questions: what mathematical modelling activity can be made to develop a rehabilitation prototype for an ankle affected by deep vein thrombosis from an engineering perspective?; and what kind of adaptations can be made to such a project to design a didactic activity that engineering students can develop? To address these questions, we chose the Anthropological Theory of the Didactic (ATD) and Didactic Engineering (DE), a robust research methodology (Artigue, 2015) that has been utilised in similar research (Bartolomé et al., 2019).

2. ELEMENTS OF ATD

The ATD proposes an epistemological model for studying human activity in its institutional dimension (Chevallard, 1999). Praxeology $[T, \tau, \theta, \theta]$ is a tool that allows the analysis of human activity. Its four components are type of task, T ; technique, τ ; technology, θ ; and theory, θ . The task is what is done; the technique is the way it is done; technology is the discourses that produce, justify, and explain the technique; theory, finally, is made up of more general discourses that produce, justify, and explain the technology. Thus, praxeology comprises two blocks, a technical-practical one $[T, \tau]$ known as know-how, and a technological-theoretical one $[\theta, \theta]$ known as knowledge.

In this theory, all human activities occur in institutions, which are conceived as stable social organisations that make activities possible because of the resources they make available to their subjects. Romo-Vázquez (2009) recognised three types of institutions, according to their relation to knowledge, as part of the training of engineers: production or research, P , teaching, T and user, U_i . The institutions of production or research include the disciplines of mathematics, M , and engineering, ED . This is where praxeologies are produced and validated. The role of teaching institutions is to transmit praxeologies to, and disseminate them among, apprentices. For example, by teaching mathematics, T_M , and engineering disciplines, T_{ED} . User institutions, U_i , host and regulate practical activities related to the use of praxeologies (e.g., workplaces, industry). Practical activities in training, such as developing engineering projects or innovations, are considered a kind of user institution since they approach, to a certain extent, professional practice. The production, teaching (dissemination), and use of praxeologies can occur in every institution. The distinctive nature of each institution depends on its predominant vocation; that is, producing, transmitting, or using mathematical praxeologies. Praxeologies can circulate among different institutions but this entails performing transformations, known as transpositive processes.

For this reason, mathematical praxeologies in engineering are often the result of transpositive processes. Moreover, they can be seen as mixed-praxeologies that contain elements of two or more institutions (Vázquez et al., 2016). For purposes of analysis, we use the Extended Praxeological Model (EPM), proposed by Castela and Romo (2011) and refined through several studies, including Peters et al. (2017), Solares et al. (2016), and Diego-Mantecón et al. (2021). Considering the mathematics institutions $-M$ and T_M- and the engineering institutions $-ED$ and $T_{ED}-$ this model is represented as follows:

$$\begin{bmatrix} T^{ed} & \tau^{ed,m} & \theta^m & \theta^m \\ \theta^{ed} & \theta^{ed} & & \end{bmatrix} \leftarrow \begin{matrix} T_M \leftarrow M \\ T_{ED} \leftarrow ED \end{matrix}$$

Where T^{ed} is an engineering type of task and $\tau^{ed,m}$ is a technique composed of engineering and mathematical elements closely interrelated and justified by two theoretical-technological blocks: one mathematical $[\theta^m, \theta^m]$, the other engineering $[\theta^{ed}, \theta^{ed}]$. For example, a technique in which the stiffness matrix is used has two validations: an engineering validation related to stiffness and material properties, and a mathematical validation associated with the validity of the matrix model based on linear algebra. The formal mathematical validations may not appear explicitly in some institutions that teach engineering (Castela & Romo, 2011; Faulkner et al., 2020).

2.1. The paradigm of questioning the world

The paradigm of “questioning the world” is characterised by the study of open researchable questions, Q , through Study and Research Paths (SRPs) (Chevallard, 2015). SRPs are teaching devices in which students, X , and the set of guides and teachers, Y , undertake investigative processes to determine a response denoted by R^\heartsuit . This response is significant in relation to certain institutions, I . The inquiry process usually begins with the study of questions derived from Q that make it possible to identify existing responses, R^\heartsuit , in works, W – research papers, web pages, consulting experts, etc. – and new questions, Q_i . The Herbartian scheme represents this process:

$$[S(X; Y; Q) \rightarrow \{R_1^\heartsuit, R_2^\heartsuit, R_3^\heartsuit, \dots, R_n^\heartsuit, Q_{n+1}, \dots, Q_m, W_{m+1}, \dots, W_p\}] \rightarrow R^\heartsuit$$

Where S represents the didactic system made up of the students and teachers who study a generating question, Q . The milieu that allows the elaboration of R^\heartsuit is made up of the sub-questions, the works studied, the existing answers found, and the relationships among these elements. In shaping the milieu, the media (mass media, websites, conferences, books, classes) play a fundamental role. The analysis of the SRP is performed through dialectics that show the process of studying the question, Q . Our current research considers two dialectics: questions and answers, and *milieu-media*. The dialectic of the questions and answers is evidenced by identifying the questions considered and the answers, both the ones identified and those produced by students. The *milieu-media* dialectic refers to the construction of knowledge that is put to the test during an SRP (Costa et al., 2015).

3. THE STUDY

This research constitutes a case study (Stake, 2010) shaped by the design of the SRP called ‘Rehabilitation Device for Deep Ventricular Thrombosis’ and its implementation in the School of Engineering Sciences and Technology (ECITEC) at the Valley of the Palms campus of the Autonomous University of Baja California. We worked with a group of 22 fifth-semester industrial design students and five seventh-semester mechanical engineers, who acted as advisors for the construction of the prototype and the calculations involved.

The SRP was designed by four researchers, one biomechanical engineer and three mathematics education researchers, two of whom oversaw implementation with the students, though all four were involved in analysing the students' work. Initially, three researchers analysed the data individually, they then compared their analyses, and finally, all four analysed the data together.

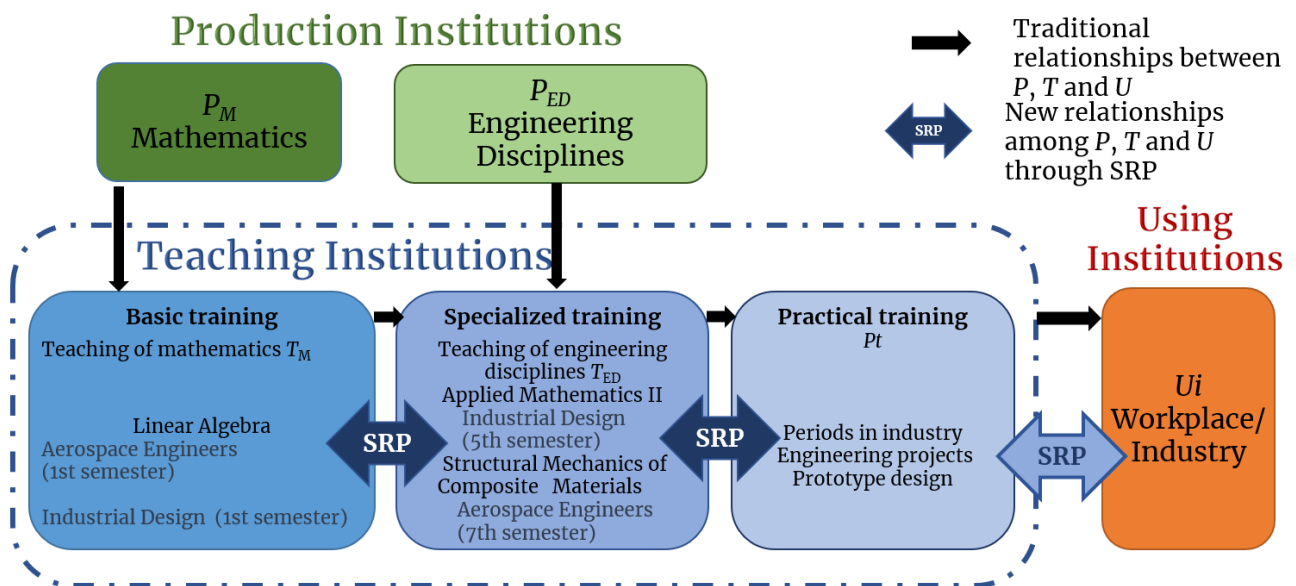
3.1. Research methodology

The research methodology chosen was inspired in didactic engineering (Artigue, 2015) because it offers a solid route for designing and implementing didactic situations in the classroom and has been adapted to design SRPs by García et al. (2019). Based on this, we defined four phases for conducting our study: 1) Preliminary analysis; 2) SRP design and *a priori* analysis; 3) Experimentation of the SRP and *in vivo* analysis; and 4) *a posteriori* analysis of the SRP implemented.

1) Preliminary analysis. In this phase, we first analysed the training model used at the ECITEC. We identified that there are basic courses (in mathematics and physics) that are compulsory to accede to the specialised level of training (engineering courses) and that both are required to reach practical training (see black arrows in Figure 1). Mathematicians or physicists give the first-level courses, but they rarely know why certain math and physics topics are taught (the *raison d'être*). The second and third levels are given by engineers who only rarely are aware of how math and physics are taught. These means that there are gaps or disconnections among these three training levels. Several research studies have highlighted this didactic phenomenon (e.g., Faulkner et al., 2020; Gonzalez-Martin, 2018). To identify the kinds of mathematics and, more specifically, the types of mathematical models that are recognised as necessary in specialised teaching, two surveys were designed, one aimed at students, the other at the professors who teach the specialised subjects. Of the 67 students from the semester 3 to 7 who responded, we found that the subjects designated as most useful are calculus and linear algebra. In addition, 54% of students said that they had worked with mathematical models. As for the professors, those who teach the subjects of Structural Mechanics of Composite Materials, Integral Calculus, and Linear Algebra were surveyed; 92% recognised the use of matrices in their courses. We then considered analysing two engineering courses that have common topics about stress calculation that require matrix models: Structural Mechanics of Composite Materials and Applied Mathematics II. The first is part of the university's specialised training for future Aerospace Engineers. It is usually taught in the seventh semester. The second is part of the specialised training in the study program of Industrial Design. It is given in the fifth semester. In order to generate new relationships between these two forms of training (see blue arrows in Figure 1), we had to include in this analysis the Linear Algebra course given in the basic training of both programs.

Figure 1. Engineering training viewed as a set of relationships among P, T, and U institutions

Training of future Aerospace Engineers and industrial designers as seen through the institutions of production, teaching, and practice



The analysis of the courses was performed by examining the textbooks and attending some classes. In particular, we identified the matrix models used in spring systems, and interviewed engineering professors to determine the role of matrix models and identify students' difficulties. One professor explained during an interview that spring systems were very useful in designing prototypes and could even be used to create a rehabilitative device for himself as he was suffering from deep vein thrombosis. This affirmation led us to delve deeper into the subject and analyse the praxeology that we call: rehabilitation device for deep ventricular thrombosis (RDDVT-P), and then examine it from an engineering perspective. We worked jointly with this professor, a biomechanical engineer, to analyse how an engineer realises this praxeology.

The rehabilitation device for deep ventricular thrombosis praxeology (RDDVT-P). The type of task consisted in designing a rehabilitation device for deep ventricular thrombosis. It can be performed following a technique composed of four main steps.

Step 1: Identify the movement features that the rehabilitation device needs to perform: flexion, extension, adduction, abduction, and circumduction, paying particular attention to the angles for each movement. The flexion movement is 20-30 degrees, the extension movement is 37-45 degrees, adduction is 22-36 degrees, and abduction is 15-25 degrees.

Step 2: Determine the degrees of freedom of the patient and prototype. In this case, it was determined that the patient had only 5 degrees of flexion and extension movement.

Steps 1 and 2 can be performed through anthropometry. The prototype can be designed for a specific person. In this case, it is necessary to determine the range of motion of the prototype, taking into account the person's range of movement. For example, we considered the case of a university professor (Professor A) who had 5 degrees of flexion and extension movement.

Step 3: Analyse the resistance of the prototype materials (wood, springs, PVC tubes, etc.). This step is divided into two parts to calculate the material stress. First, to determine the appropriate material it was necessary to obtain exact weight of the professor's affected leg. The professor suggested he be weighed while sitting down since the prototype would be used in that posture, and that would determine the force. Then we had to calculate the area of the foot of the prototype's user. Next, we calculated the stress using $\sigma = F/A$, where F is the force, A the area, and σ the stress. After that, we evaluated the mechanical properties of different kinds of wood to find one that could handle the stress applied by the user and considering cost. That is how we obtained the material.

For the spring, we used Hooke's law which states that $F = KU$, where F is the force vector, K the stiffness matrix, and U the displacement vector. It was necessary to build the stiffness matrix, then we could introduce the initial conditions for the force applied and measure the displacements. The initial conditions of the study problem were as follows:

- The subject is a 32-year-old patient recovering from "deep venal thrombosis".
- The doctors recommend beginning with motor rehabilitation due to the affectation of the professor's left leg two weeks after the vascular event.
- The decision was to do rehabilitation 3 hours per week and monitor the subject's muscular reactions due to the possible accumulation of lactic acid and cramping.

One additional condition was that the subject had no strength in the foot from the ankle joint downwards. Testing performed while he was in a seated position with the soles of both feet fully supported. He was asked to raise the toes of the affected foot in dorsiflexion but was unable to do so. He was instructed to execute this type of movement with help to reduce foot drop syndrome and strengthen the muscles affected by deep vein thrombosis.

The next activity consisted in calculating the stress required for the spring. This meant considering the following conditions in order to apply Hooke's law:

1. Make a free body diagram.
2. Determine the location/s where stress would be applied.
3. Apply Hooke's law.
4. Solve the equations.
 - 4a. Determine the simple tension stress.

For Hooke's law to be applicable to the problem, the following conditions had to be met:

1. The longitudinal appendage being analysed had to be straight.
2. The axial load had to be applied at the centre of the appendage.
3. The section analysed had to be uniform.

First, we calculated the measured, or experimental, stiffness of a parallel acting spring system $K = \frac{F}{X}$, where:

K = stiffness

F = force

X = displacement

$$K = 19.9146 \frac{N}{m}$$

Then we calculated the shear stress: $T = \frac{F_c}{A_c}$

T = shear stress

F_c = force applied

A_c = tube area

Mass was taken from ergonomic tables: $M = 2 \text{ kg}$, $M = F_d$

$$F = (2 \text{ kg})(9.81 \text{ m/s}^2)$$

$$F = 19.62 \text{ N}$$

M = momentum

d = distance

F = force

$$M_1 = (19.62 \text{ N})(0.04 \text{ m}) = 0.7848 \text{ Nm}$$

$$M_2 = (19.62 \text{ N})(0.10 \text{ m}) = 1.962 \text{ Nm}$$

The areas for the tube, $A_1 = \pi(d)^2$, $A_2 = \pi(D)^2$, were:

d = interior diameter

D = outside diameter

$$A_1 = \pi(2.4)^2 = 18.1 \text{ mm}^2$$

$$A_2 = \pi(5.75)^2 = 103.9 \text{ mm}^2$$

and $A_c = A_2 - A_1 = 85.8 \text{ mm}^2$

At that point, we proceeded to construct the matrix

$$\begin{pmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{l} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12EI_y}{l} & 0 & 0 & 0 \\ 0 & 0 & 0 & GI_{GG} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4EI_y}{l} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{4EI_z}{l} \end{pmatrix}$$

where

l = spring length

E = modulus of elasticity

G = body modulus

I = inertia

Upon calculating the shear stress and momentum of inertia we obtained:

$$\begin{pmatrix} 241.44 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.81 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.46 * 10^{-4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.67 & 0 & 0 \\ 0 & 0 & 0 & 0 & 241.44 & 0 \\ 0 & 0 & 0 & 0 & 0 & 241.44 \end{pmatrix}$$

We then used tables to find the values for the modulus of elasticity, body modulus, and permissible torsional stress. We identified the suitable spring and installed it on the prototype.

Step 4: Useful life; this process was analysed by a bioengineer, a member of the research team who proposed the following solution. We began by calculating the flexion (σ_x) and torsion (τ_{xy}). $\sigma_x = \frac{M \cdot d}{I}$, $\tau_{xy} = \frac{T \cdot d}{J}$. where:

σ_x is the normal stress in direction x

τ_{xy} is the torsional tension or tangential stress on the x -face and in direction y

M is the bending moment of the critical section

I is the moment of inertia of the transverse axis

J is the polar moment of inertia of the axis

Then, using the theory of maximum shear energy, we obtained

$$d = \left[\frac{32 n}{\pi S_y} (m^2 + T^2)^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

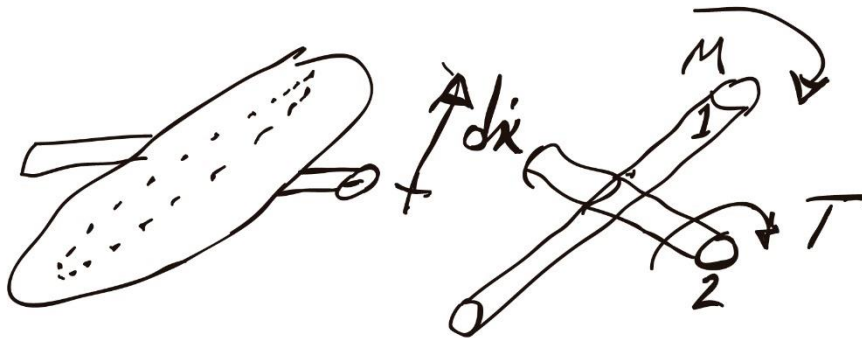
where $S_{sy} = S_y/2$ and $n = \frac{S_{sy}}{\tau_{max}}$

For dynamic loads, Sines' theory states that the bending fatigue strength does not vary due to the existence of an average torsional stress until $\tau_{max} = 1.5 S_{sy}$. According to Soderberg,

$$n = \frac{\pi d^3}{32 \sqrt{\left(\frac{T}{S_{sy}}\right)^2 + \left(\frac{M}{S_{se}}\right)^2}}$$

And by the theory of maximum shear energy, we obtain $d = \left[\frac{32n}{\pi S_{sy}} (m^2 + T^2)^{\frac{1}{2}} \right]^{\frac{1}{3}}$ where S_{sy} is the allowable stress and S_{se} is the elastic limit stress. For the prototype's structure (Figure 2), we considered that when element 1 works in the flexural movement of the foot it does so in the form of the moment, and that element 2 works torsion, as shown in Figure 2.

Figure 2. The prototype's structure was devised by the biomechanical engineer



The calculations obtained for the useful life of the prototype using Soderberg over Goodman gave greater reliability because it works with the maximum volume theory, which implies that we will have larger areas and, therefore, a higher safety factor.

Transposition of engineering praxeology to school praxeology. The engineering praxeology was then transposed to a school praxeology. The **type of task** (T^{ed}) was maintained; that is, design a rehabilitation device for deep vein thrombosis of the ankle. For this step, the students were asked to research existing rehabilitation devices in order to understand how they could design their own prototype, but without copying any device they saw. The **technique** ($\tau^{ed,m}$) involved several steps: (1) determining the movements that the prototype had to perform; (2) determine suitable materials; (3) selecting a spring to help the subject make the movements with some resistance; and (4) assembling the prototype. The shear stress for the materials and springs are calculated using Hooke's Law and a matrix model $F = KU$ (F : Force Vector, K : Stiffness matrix, U : displacement vector), to determine the correct materials and spring. **Technology** (θ^{ed}, θ^m). Since this is a matrix model, it is necessary to know how to build the stiffness matrix in order to determine the stresses that the materials and springs had to resist. The **Theory** (θ^m, θ^{ed}), finally, is based on linear algebra and the resistance of materials.

Step 4 of the engineering praxeology was not considered in the school praxeology because the topics related to its realisation are covered in more advanced courses.

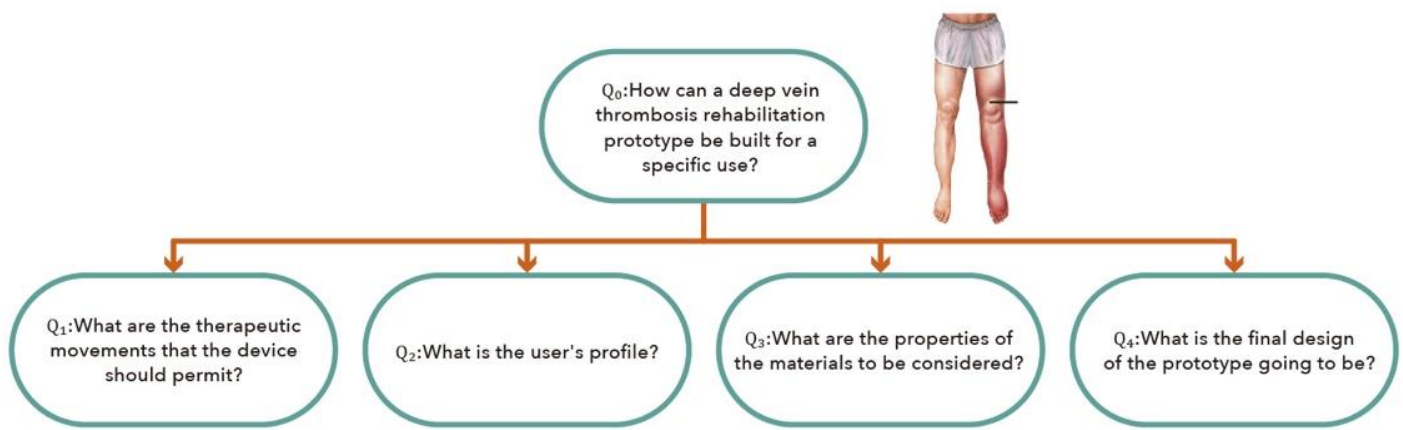
2) Design of the SRP “Rehabilitation device for deep ventricular thrombosis” and *a priori* analysis. Considering that the SRP would be developed by students starting their third year of university, it was necessary to think, more in terms of a prototype that consisted of plates and springs than in a structure. Likewise, we had to consider the types of calculations they would make, using matrix models which the students had just seen in their course on linear algebra. This knowledge was recent, but they could work on stress calculations since they were only asked to calculate the inverse of a matrix in order to obtain the stiffness matrix and then make a product of the stiffness matrix with the displacement vector, giving an answer to the effort required for the spring to be used and the effort of the proposed material. Therefore, the proposal was to design a rehabilitation device for deep ventricular thrombosis as a system of springs to aid in the rehabilitation of people suffering from this condition so they could recover mobility in the ankle of an affected leg. The generating question Q_o was: *How to build a deep vein thrombosis rehabilitation prototype for a specific user?*

According to ATD, the study of the question Q_o is related to the construction of the RDDVT rehabilitator’s school praxeology, which is associated, in turn, with specific derived questions, Q_i , existing answers, R_i , and the *medias* and *milieu* available. We identified a two-step technique: design the rehabilitation device prototype RDDVT (Q_1 - Q_4 and derived questions), and choose the material for the spring and perform the stress calculations (Q_2 , Q_3 - Q_4 and derived questions). Students had to find a way to execute these two steps. For the first one, they had to investigate existing types of ankle rehabilitation devices and choose one. For the second, they had to investigate the types of springs that exist, choose one, and calculate the stresses to which the device would be subjected using Hooke’s law, as shown in the engineering praxeology in the previous section. Step 2 required that students understand and apply Hooke’s law and determine –or investigate– how to build and calculate the stiffness matrix for the problem they were being asked to solve. The students could use such computer programs as MatLab, Scilab, and SolidWorks (*medias*) to perform the necessary calculations.

3) Experimentation of the SRP and *in vivo* analysis. The students were organised as follows. At the beginning of the semester, the SRP “Rehabilitation Device for Deep Ventricular Thrombosis” task was proposed to the industrial design students in the course applied mathematics II. Teams of four were formed. Work was reviewed every two weeks, and at a certain point the decisions was taken to invite seventh-semester aerospace engineering students to participate. Those students were taking a course on the structural mechanics of composite materials where they analysed numerous different problems where the stress for other structures and materials was calculated using Hooke’s law. One of those students was assigned to each team to help them with any questions. The professor of the course on structural mechanics of composite materials advised the students on aerospace engineering. This led us to imitate the organisation of engineers at the workplace,

various specialisations, and different levels of experience (Kent & Noss, 2002). The project lasted sixteen weeks. Two interim reports were requested, the initial one in week eight. The final report was turned in week sixteen. The students had access to the university's workshops under the supervision of academic technicians. The professor who required the prototype could be called in to perform measurements and test proposals so that the students could see the progress of their prototype. The formulated and derived questions that were studied are schematised in figures 3A-E.

Figure 3A. A pictorial description of the derived sub-questions from Q_0



The first three sub-questions in Figure 3A are related to the first three steps involved in developing the praxeology for the rehabilitation device for deep ven-tricular thrombosis. The sub-question Q_4 is a reformulation of Q_0 , but it considers the products of the study and the research conducted for sub-questions Q_{1-3} .

For the questions shown in Figure 3B, participants studied the characteristics that the device should have, based on general knowledge about deep vein throm-bosis. Regarding the questions in Figure 3C, students tried to analyse specific, rel- evant characteristics of the patient.

Figure 3B. The sub-sub-questions from sub-question Q₁

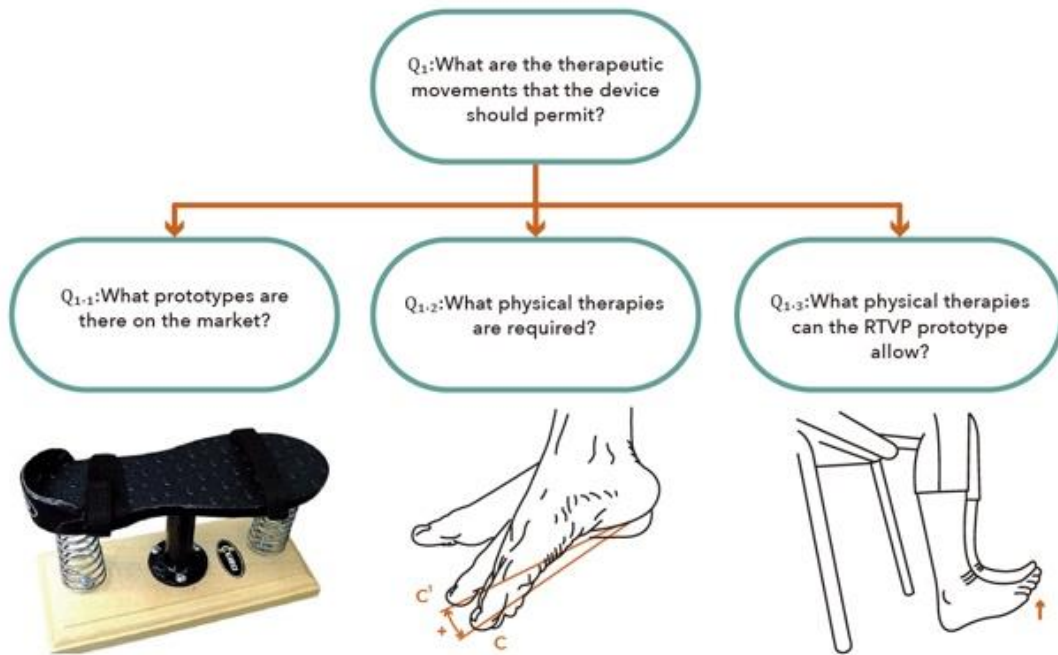


Figure 3C. The sub-sub-questions derived from sub-question Q₂

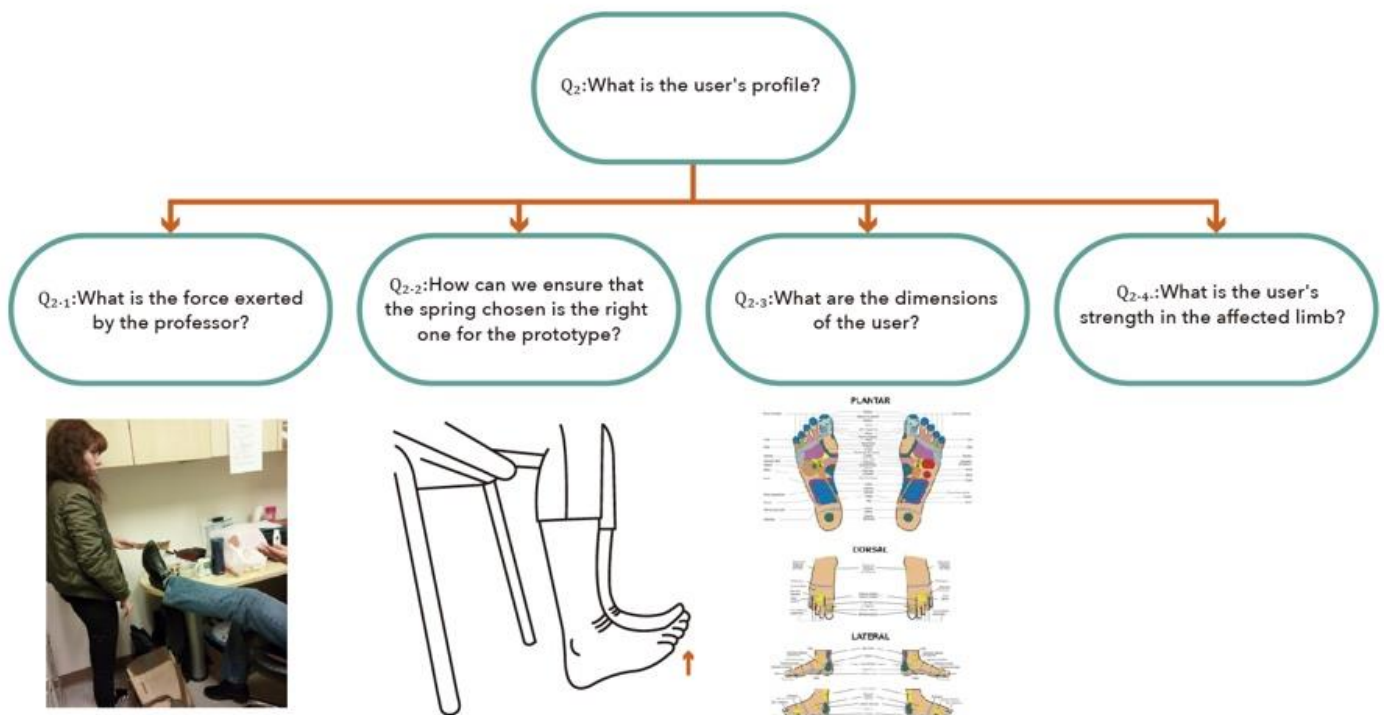
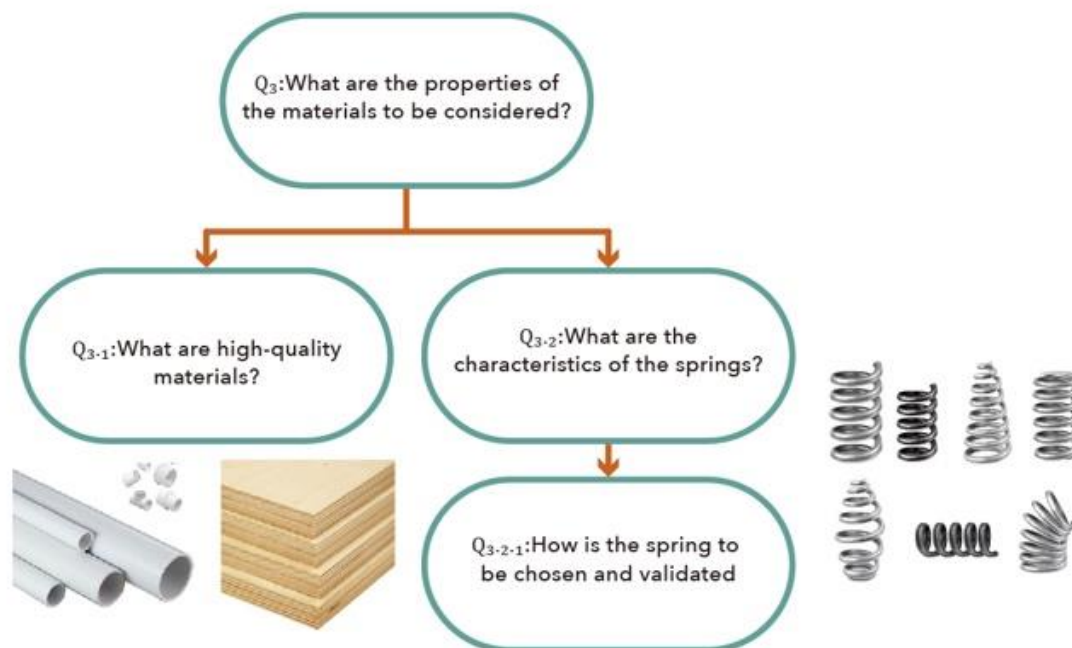


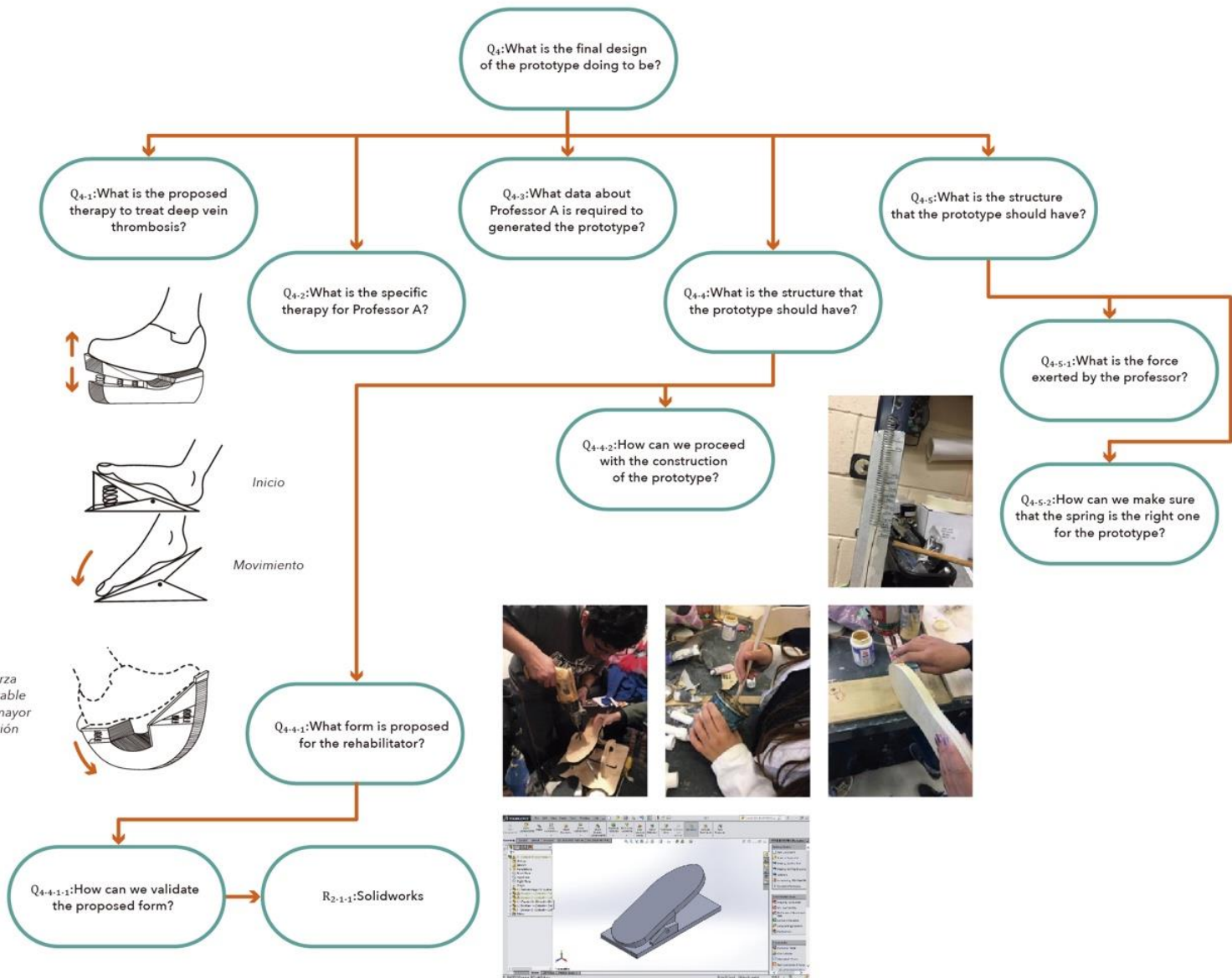
Figure 3D addresses the characteristics of the materials to be used, mainly surfaces and springs. The knowledge constructed by the students should suffice to propose a design for the prototype, guided by the sub-questions in Figure 3E.

Figure 3D. The sub-sub-questions derived from sub-question Q₃



4) *A posteriori* analysis. This analysis presents the students' work related to constructing the school praxeology in terms of "selecting the spring and making the stress calculations" as constructed by the students to answer question Q₀, mainly to ensure the strength of the chosen or composite constructed material. To this end, we analysed the students' partial and final reports and identified the question-answer dialectics. This allowed us to elucidate the study and research process followed to elaborate the prototype and the school praxeology developed by three teams of students.

Figure 3E. The sub-sub-questions derived from sub-question Q₄



4. RESULTS

In this section, we present the work developed by the students, considering teams 1, 2, and 3. We refer mainly to their partial and final reports to reconstruct the SRP they followed. At the first moment, the students were introduced to the task, T^{ed} ; that is, build a prototype to rehabilitate people who suffer from deep venal thrombosis in the leg (RDDVT). A two-step technique was proposed: first, design the rehabilitation device prototype, RDDVT (Q_1 - Q_4 and derived questions), and, second, select the materials and spring and make the necessary stress calculations (Q_2 , Q_3 - Q_4 and derived questions). To make these steps greater precision, the device was designed for a specific person who had this disease, a professor at the university where the SRP was implemented. This measure allowed the students to construct a specific prototype and to achieve theoretical and practical validations for it.

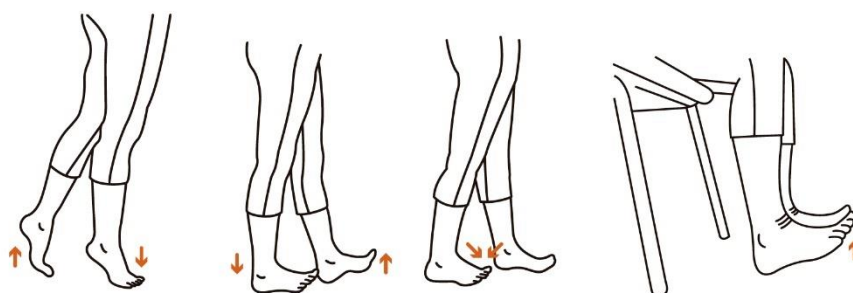
4.1. Design of the prototype of the rehabilitation device, RDDVT

First, the students investigated the characteristics of deep vein thrombosis, then they analysed the types of exercises that are necessary for rehabilitation. To address the question Q_4 , how is the prototype to be designed? They addressed four derived questions:

- $Q_{4,1}$. What is the therapy proposed for treating deep vein thrombosis?
- $Q_{4,2}$: What is the specific therapy that Professor A requires?
- $Q_{4,3}$: What data about Professor A is required to generate the prototype?
- $Q_{4,4}$: What structure should the prototype have?

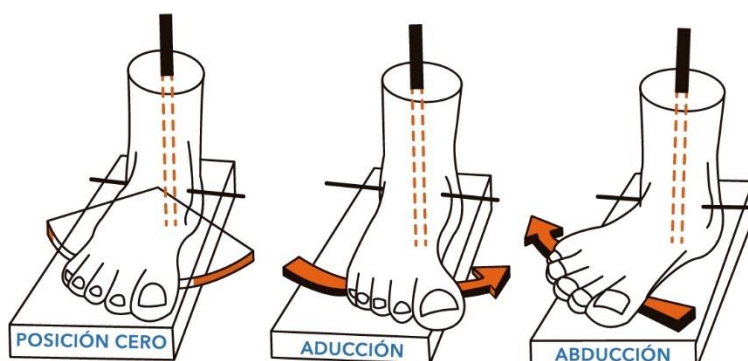
All three teams presented approximately the same therapy for deep vein thrombosis, $Q_{4,1}$. Team 1 reported that therapy consists of four movements: walking on the ankles and toes, standing with the feet apart while maintaining a fixed distance between the shoulders, lifting the toes, and balancing on the heels (Figure 4). To perform these movements patients must lean on furniture or walls so they do not lose their balance. For this reason, rehabilitators are more comfortable and effective.

Figure 4. Walking on the ankles and toes, standing with the feet apart while maintaining a fixed distance between the shoulders, lifting the toes, balancing on the heels, flexion/extension and adduction/abduction movements



Team 2 also emphasised flexion/extension movements and adduction/abduction movements in its report, but paid particular attention to the latter (Figure 5).

Figure 5. Description of adduction/abduction movements



Team 3, in contrast, highlighted circumduction for physical therapy and designed their prototype in accordance with that movement. To address $Q_{4.2}$: what is the specific treatment for Professor A? Team 3 met with the user to identify his particular needs. They recognised that therapy should consist of a pendular movement that the user could execute for 30 minutes a day at home or at work. They proposed a pedal that can be put on and taken off as a solution. They took foot measurements and determined that they had to carry out an anthropometric study of the user's foot. Teams 1 and 2 also interviewed the user to identify the same movements as Team 3, but they studied pedals available on the market. Team 1 analysed three models, one of which motivated them to choose a spring system and improve the aesthetics and comfort of the device (Figure 6).

Figure 6. One of the pedals analysed by Team 3



Team 2 found that most of the ankle rehabilitation devices available on the market are of the machine car pedal (MCP) type, but that they do not permit all the movements required for rehabilitation or contemplate the full range of movement. Hence, they cannot be adapted for complete, personalised therapies.

To answer $Q_{4.3}$: what data about Professor A is required to generate the prototype? The 3 teams interviewed the professor and obtained the following data:

Gender: male

Age: adult, 32-50 years old

Occupation: Assistant Director of the Faculty

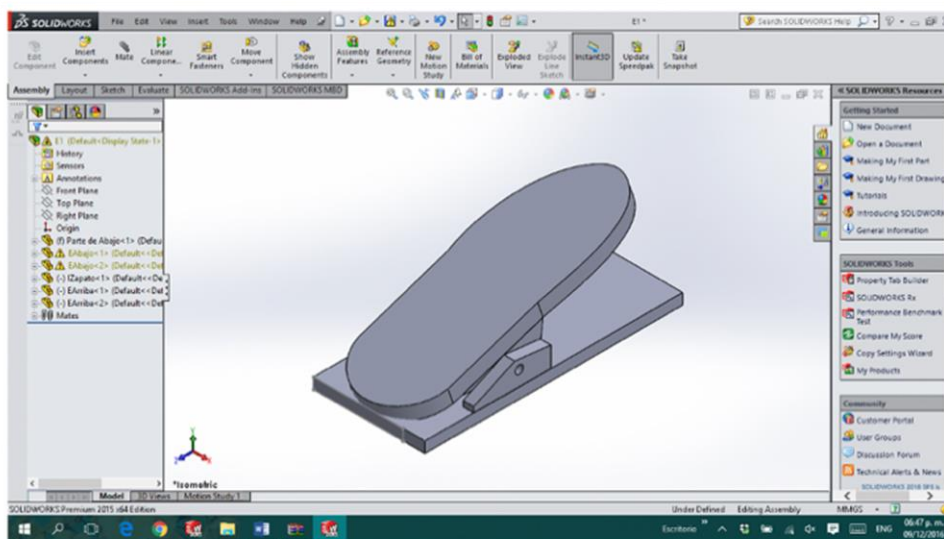
Shoe size: 27 Mexican, 9 American

Foot size: approximately 30 cm long by 10 cm wide

But Team 3 was the only one that measured the force the user had in the affected foot using a dynamometer. The result was force 0 for flexion. They took this data into account in designing their prototype, which resulted in a distinct design that we examine below.

Q_{4,4}: what is the structure that the prototype should have? The students identified that the basis for the structure of the prototype was their analysis of treatment for the disease, so they went on to questions Q_{4,1} and Q_{4,2}. After analysing the results, Team 1 designed a prototype that only allowed the professor to make one of the five movements solicited in the project: flexion. They made the calculations using Solidworks software. This team made several proposals in Solidworks to see which one solved the problem in the best way. After running simulations, they chose the one shown in Figure 7.

Figure 7. Proposal of a prototype in Solidworks



4.2. Choose the material and spring, and make the stress calculations

One key question was Q_{4,5}: what type of spring should be used in the prototype? The students addressed two derived questions:

Q_{4,5,1}: What is the force exerted by Professor A?

Q_{4,5,2}: How can we make sure that the spring is the right one for the prototype?

Concerning question Q_{4,5,1}, Team 1 did not take into account the force that Professor A could apply. Once the proposal was made on Solidworks, those students went on to print the blueprints to be able to trace the exact measurements of the components on the wood. Once the pieces were marked, they cut them and gave them the required inclinations. They did this with the help of the sander because the pieces needed to have an exact form, and the machines could not perform that process satisfactorily. Once they had all the pieces, the students incorporated the bolt that crosses and joins the two parts of the pedal. To place the spring, they decided on the exact points, drilled a hole, inserted the spring, pressurised it, then fastened it with a pneumatic stapler. Finally, they painted the device and added an anti-skid covering so that the professor's foot would not drag the pedal around. Figure 8 shows the finished prototype.

Figure 8. Design of Team 1's prototype



Concerning questions $Q_{4,5,2}$, the students used the Solidworks program to obtain the stress for a suitable spring for the pedal, then they calculated the precise location for the spring. This option impeded determining how to carry out the calculations, so they did not know where the values for stress or the properties of the material came from. Those activities were performed automatically in Solidworks, which meant that it was used as a black box. Finally, this prototype only allowed the professor to make the flexion movement.

Team 2, in contrast, strove to see if they could develop a design that would allow all five movements, though in the end their prototype only allow four: flexion, extension, adduction, and abduction. When we asked what force the teacher could exert on the prototype, they answered with the teacher's weight, so they had not considered the fact that the professor would be sitting while using the device. Figure 9 shows Team 2's prototype.

Figure 9. Design of Team 2's prototype



Team 2 did not use a matrix model to make the calculations, as shown in Figure 10. Instead, to determine the stress for the spring used on the prototype, they used the Solidworks program, in part because they were taking the Solidworks course simultaneously and because the program facilitates the process of making the required calculations. They entered the values of the materials for their design, then had the program calculate the stress at various points to anticipate the pressure that the professor would exert on the device. That step allowed them to proceed with construction. This team did not present details on the elaboration process. Once again, the Solidworks software was used as a black box for the calculations. But it was also used to validate the pedal's shape (making it possible to print and build it) and the importance of the spring they chose to ensure that it would not deform with use.

Figure 10. Calculations for the stress on the spring

Desarrollo de problema

Debido a que el resorte se mueve en una dirección, los cálculos son para un caso de deformación un axial. Entonces, con los datos se puede encontrar el esfuerzo, la elasticidad y el Módulo de Young.

$$D1=1.3 \text{ cm y } D2=1.5 \text{ cm} \rightarrow R1=(6.5 \times 10^{-3}) \text{ m y } R2=(7.5 \times 10^{-3}) \text{ m}$$

$$A=[(R2)^2 - (R1)^2]$$

$$A=[(7.5 \times 10^{-3})^2 - (6.5 \times 10^{-3})^2]$$

$$A=0.0234 \text{ m}^2$$

$$\epsilon = \delta/L \rightarrow \epsilon=(1.96 \text{ in}-.9408 \text{ in})/(1.96 \text{ in}) \rightarrow \epsilon=0.52 \quad \text{ELASTICIDAD}$$

$$\sigma = F/A \rightarrow \sigma = (44.68 \text{ N})/(.0234 \text{ m}^2) \rightarrow \sigma=1,909.4 \text{ Pa} \quad \text{ESFUERZO}$$

$$E=(1,909.4)/0.52 \rightarrow E=3671.92 \text{ Pa} \quad \text{MÓDULO DE YOUNG}$$

Por tal razón, el Esfuerzo Lineal capaz de soportar el resorte es de 3671.92 Pascales

Team 3 did take into consideration the finding that the professor had no strength in his limb ($R_{4,5,1}$). This led them to design a prototype that allowed the professor to do flexion, extension, adduction, abduction, and circumduction movements. They also presented details on the elaboration process:

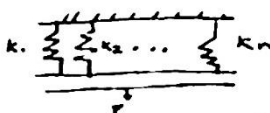
1. The team cut the wooden pieces using the professor's measurements.
2. They sanded all the pieces.
3. They painted all the wooden pieces.
4. They measured, cut, and sanded the PVC tubes.
5. Finally, they assembled the prototype.

Afterwards, they addressed $Q_{4,5,2}$. They made the calculations for the stress on the spring in a manner analogous to that of the engineering praxeology (see section 3.1). First, they determined the stiffness component, k , for the spring, then they calculated the shear stress to find the force, F , exerted by the professor's foot and calculated the area A . For this parameter, they needed to consider the value of the

mass of the teacher's foot, which they looked up in ergonomic tables. Finally, they calculated the moment of inertia, the last parameter they needed (Figure 11).

Figure 11. Image of Team 3's stress calculations.

① $k = F/x$ Rigidez medida o experimental
 $k = 19.9146 \text{ N/m}$
 $k = \text{Rigidez}$
 $F = \text{Fuerza}$
 $x = \text{desplazamiento}$
 Sistema de resortes que actúan en Paralelo



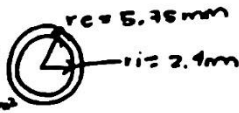
② Esfuerzo constante
 $T = \frac{F_c}{A_c}$
 $T = \text{Esfuerzo constante}$
 $F_c = \text{Fuerza aplicada}$
 $A_c = \text{Área del tubo}$

La masa se saca de tablas ergonómicas
 $M = Fd$
 $M = 2 \text{ kg}$
 (tablas)
 $F_c = (2 \text{ kg})(9.81 \text{ m/s}^2)$
 $F_c = 19.62 \text{ N}$
 Fuerza que ejerce el pie

$M = \text{momento}$
 $d = \text{distancia de aplicación de la fuerza}$
 $F = \text{Fuerza}$

$M_1 = (19.62 \text{ N})(0.04 \text{ m}) = 0.7848 \text{ Nm}$
 $M_2 = (19.62 \text{ N})(0.10 \text{ m}) = 1.962 \text{ Nm}$

Área = πr^2
 $A_1 = \pi(2.4)^2 = 18.09 \text{ mm}^2$
 $A_2 = \pi(5.75)^2 = 103.86 \text{ mm}^2$



$A_{\text{cortante}} = A_2 - A_1 = 85.77 \text{ mm}^2 = A_c$

Each parameter was determined considering the characteristics of Professor A and the conditions of the prototype's design. Following Bissell and Dillon (2000), we can see that the story of Team 3's modelling activity originated in, and was guided by, Q_0 . These students had to construct the stiffness matrix in order to obtain the stress calculation, so they had to find the values of the modulus of elasticity, E , the moment of inertia, I , the body modulus, the area, and the length of the spring (Figure 12). The process of doing the calculations themselves without the use of a computational program made these students realise that the mathematics they were learning in the classroom can actually be used in real-world tasks.

Team 3's prototype was the most complete one of the three designed because it allowed all the movements requested in the original project. What this team did not foresee was that the springs they chose expired after only a short period of use, so they had to search for replacements. During development of the SRP, they failed to consider the question: $Q_{durability}$; that is, what is the durability of the spring chosen? (related to step 4 of the engineering praxeology). At the last moment, however, they came up with a solution: they changed the spring for rubber bands which improved the operation of the prototype. They used some of the parameters from their

original calculations, but entered the conditions for the rubber bands until they found a type that was suitable for their prototype.

Figure 12. Image of the stress calculations with the matrix model.

Matriz

$$K_{eq} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{l} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12EI_y}{l} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{GI_{GG}}{l} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4EI_y}{l} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{4EI_z}{l} \end{bmatrix}$$

K_{eq} = Rigidez
 a = radio interior
 $a = 2.1mm$
 b = radio ext.
 $b = 5.78mm$
 l = longitud resorto
 m = masa
 $m = 0.050kg$

E = modulo de elasticidad
 I = Inercia
 G = modulo de cuerpo

③ Esfuerzo cortante = T
 $T = \frac{F_c}{Ac}$
 $F_c = F_1 + F_2 = 1962N$
 $F_c = 89.24N \approx 40N$
 $T = \frac{40N}{0.8577 \times 10^{-4} m^2}$
 $T = 0.004663635$

$l m^2 = 1000000 mm^2$
 diferencia = $8577 mm^2$
 de areas = $0.8577 \times 10^{-4} m^2$

$m = 0.050kg$
 a = radio int.
 $a = 2.1mm$
 b = radio ext.
 $b = 5.78mm$
 l = longitud de resorto

Momento de inercia
 $I = Inercia$
 $I = m \frac{(3a^2 + 3b^2 + l^2)}{12}$
 $I = 0.4198 kg/m^2$
 Momento de inercia en ejes Y, Z

④ Matriz

$$K_{eq} = \begin{bmatrix} 241.4425 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.8121 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.4615 \times 10^{-4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.6706 & 0 & 0 \\ 0 & 0 & 0 & 0 & 241.4425 & 0 \\ 0 & 0 & 0 & 0 & 0 & 241.4425 \end{bmatrix}$$

$x = 20mm$
 $y = 10mm$
 $z = 20mm$

It is important to note that Professor A is still using this prototype (Figure 13). However, we have not performed tests or taken measurements to determine if his condition has improved. These are questions for another study.

Figure 13. Design of Team 3's prototype



Implementing this SRP with linear algebra students introduced them to a vision of mathematical modelling in which math is combined with other kinds of knowledge to address exciting questions and provide solutions to real problems.

5. CONCLUSIONS

In this experience, we can appreciate how the students were involved in solving a specific, real problem, more closely linked to practice than theory. For this reason, the conceptualisation of the paradigm of questioning the world, and the use of study and research paths, generated a valuable tool for motivating, analysing, and visualising the students' actions. They addressed mathematical topics that seemed obligatory since the activity was carried out in the math classroom, but they also studied and researched other aspects of developing a helpful device, such as mechanics, physical therapy, and computer modelling. This led the students to connect a first semester university course with courses in more advanced semesters. Almost surely, they will be more aware of the role of mathematics when they take those later courses. Although mathematics can remain hidden in some engineering tasks (Williams & Wake, 2007), experiencing it in a math classroom will prepare students to make it emerge when a practical problem they are trying to solve requires it. Therefore, the main contribution of this study to educational research in mathematics is that it presents the analysis of the implementation of a didactic device that was proposed to fill the gap between the teaching of mathematics and instruction in engineering. Through the question-answer dialectic, this analysis revealed the processes that students followed to construct a mixed-praxeology in response to generating-questions.

The students studied the finite element method (white box) but opted to use computational tools like Solidworks (black-grey boxes) to model the product they wanted. That procedure contrasts to what usually occurs in math classrooms, where the importance of white boxes predominates. In the present case, the students were free to make these kinds of choices, as usually happens in the questioning the world paradigm. Since the project assigned was an eminently practical

engineering activity, considering the notion of a mixed praxeology was almost mandatory in a study framed in the ATD. Other instructional designs for math classrooms can be developed by defining an engineering-type task, T^{es} , such as developing a prototype for rehabilitation, proposing it in a way that makes it relevant to the classroom, and applying a mixed technique, $\tau^{m,es}$, that contains elements of both math and engineering. This approach can be helpful for both teachers interested in promoting inquiring activities and researchers who wish to enrich their empirical knowledge of this branch of educational research.

The rehabilitation device developed for the deep ventricular thrombosis praxeology is the answer to the first research question proposed in this study; that is, what mathematical modelling activity is required to develop a rehabilitation prototype for an ankle affected by deep vein thrombosis from an engineering perspective? The tools provided by the ATD allowed us to analyse this question and adapt it as a didactic activity. We think that presenting this process in a way that engages students in a problem that is close to their reality is an empirical contribution that must be considered in the field of educational research.

More research needs to be done to implement the study and research paths in the engineering mathematics classroom, but they constitute a promising strategy for attracting students to activities that genuinely engage them in engineering problems. The ultimate goal is to achieve that the questioning the world paradigm becomes an ideal place for interaction between professors and students in the mathematical training of future engineers.

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Design of a rehabilitation device for thrombosis: a mathematical modelling activity in the training of engineers

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This paper addressed the didactical phenomenon identified in further research: the failure of connection between mathematics and engineering courses in the training for future engineers. Two research questions oriented to this study: what mathematical modelling activity can be made to develop a rehabilitation prototype for an ankle affected by deep vein thrombosis from an engineering perspective?; and what kind of adaptations can be made to such a project to design a didactic activity that engineering students can develop? Based on some elements of ATD-theory, we propose analysing mathematical modelling from an engineering perspective and transposing it to mathematics teaching through didactical devices: the Study and Research Paths (SRP), that promote inquiry in the classroom. To build these activities, we chose a methodology inspired by didactic engineering, which offers a solid route for designing and implementing didactic activities in the classroom, following four phases: 1) Preliminary analysis, 2) SRP design and a priori analysis, 3) Experimentation and in vivo analysis, and 4) A posteriori analysis of the implemented SRP. In the first phase, we analyse how to build a rehabilitation device for deep ventricular thrombosis from an engineering perspective, obtaining a referent about mathematical modelling in engineering activity. In this case, the stiffness matrix model is associated with a spring system. Then we perform a transposition, maintaining the engineering task and adapting the technique related to use of mathematical models. Thus, the SRP proposed was to build a rehabilitation device for deep ventricular thrombosis as a system of springs to help rehabilitate people who suffered from thrombosis to recover mobility in the ankle of an affected leg. In the third phase, the students were organised into teams. The project lasted sixteen weeks. Two interim reports were requested, the initial one in week eight. The final report was turned in in week sixteen. The students had access to the university's workshops under the supervision of academic technicians. The professor who required the prototype could be called in to perform measurements and test proposals so that the students could see the progress of their prototype. In the fourth phase, a posteriori analysis of the implemented SRP was made. We elucidated the study and research process followed to realise the prototype and the school praxeology developed by three teams of students.

We concluded that SRP proposed to students and how they developed it allowed us to answer the two research questions. Using ATD notions, it is possible to transpose mathematical modelling activity in engineering to school. In this case, the students did not realise step four developed in the engineering perspective. However, they made a rehabilitator device that a person uses. The students that produced the best rehabilitator had a clear idea of the role of the mathematical model in this project. They made all calculations to guarantee its resistance. They related different types of knowledge: mathematical, engineering, and practice come from different courses and the investigations made for this project.