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Abstract

Number sense is regarded as a strong predictor of mathematics achievement at formal school. This paper describes the improvement of early math skills using computer-assisted instruction and the effectiveness of an intervention program. An experimental design with control and experimental groups, and pre- and post-intervention measurement was used. Participants were 48 preschool students (aged from 4.91 to 5.91; 21 were males and 27 were female), whose relational and numeracy skills were assessed before and after training. Differences between groups were analyzed. Participants were students from four schools (two public and two private). Schools were in middle-class neighborhood, in 100.000 inhabitants towns. Descriptive analysis, discriminant analysis and hypothesis test were calculated. Null hypothesis of equality between groups was rejected (Wilks' $\Lambda = .468$; $X^2 = 31.46$; $p < .001$). Therefore, the difference between the experimental and control groups was statistically significant. Also, discriminant analysis indicated that 83.3% of students were classified correctly in their group. Significant differences between control and experimental group in classifications ($p = .001$), correspondence ($p = .001$), counting structured ($p = .001$) and resulting ($p = .000$) skills, were found. Results supported the effectiveness of the intervention program, and indicated math skills that were significantly improved

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by training: classifications and correspondence as relational skills, and counting structured and resulting as numerical skills. Educational implications and future lines of action are discussed.

Keywords: mathematics skills, remedial programs, preschool education, numeracy, computer-assisted instruction, mathematical achievement.

Resumen

El sentido numérico es considerado como un fuerte predictor del rendimiento matemático en la escuela formal. El presente trabajo plantea el perfeccionamiento de las habilidades matemáticas tempranas mediante el uso de la enseñanza asistida por ordenador. Se utilizó un diseño experimental, con grupo control y medidas pre- y pos-intervención para estudiar la eficacia de la intervención en una muestra de 48 alumnos de educación infantil, y las diferencias existentes en las habilidades relacionales y numéricas tras la implementación del programa computerizado. Del total de la muestra 21 alumnos fueron niños y 27 niñas, cuyas edades oscilaban entre los 4.91 y los 5.91 años. Se emplearon análisis descriptivo, discriminante y contrastes de hipótesis como técnicas de análisis de los datos. Los alumnos pertenecían a cuatro centros educativos, dos de ellos públicos y dos concertados. Los centros estaban situados en zonas urbanas de clase media en poblaciones de unos 100.000 habitantes. Se rechazó la hipótesis de igualdad entre los grupos ($Lambda$ de Wilks = 0.468; $X^2 = 31.46$; $p < 0.001$), pudiendo concluir que la diferencia entre el grupo experimental y control era estadísticamente significativa. Asimismo, el análisis discriminante confirmó que el 83.3% de los alumnos fueron clasificados correctamente en su grupo. Se muestran diferencias significativas en los resultados de clasificación ($p < .001$), correspondencia ($p < .001$), conteo estructurado ($p < .001$) y resultante ($p < .001$) entre los grupos experimental y control, avaladas por el tamaño del efecto. Los resultados obtenidos apoyan la eficacia de la intervención e indican las habilidades que se ven favorecidas en mayor medida por el entrenamiento: clasificaciones y correspondencia en el ámbito relacional, y conteo estructurado y resultante en el ámbito numérico. Se discuten las implicaciones educativas y las futuras líneas de actuación.

Palabras clave: habilidades matemáticas, programa de recuperación, educación infantil, aritmética, enseñanza asistida por ordenador, logro matemático.

Introduction

Early math skills, frequently grouped together under the construct, «number sense» (Berch, 1998; Hannula, Lepola, & Lehtinen, 2010), are

developed earlier than formal mathematical abilities. They are a cornerstone which more complex mathematical knowledge is built upon. An example of this is preverbal number sense, which refers to features such as subitizing (Le Corre, Van de Walle, Brannon, & Carey, 2006) and which comes before counting, or the development of an estimated representation of magnitude. This comes about before numerical representations (Lipton & Spelke, 2005). Number sense allows children to recognize different quantities and estimate quantities, allowing them to not only succeed in school, but also solve problems in everyday life.

Despite the importance of this concept and its significance, there is no general agreement at present on the conceptualization and operationalization of number sense (Gersten, Jordan, & Flojo, 2005). Researchers agree that it involves a set of related number and operations skills from the age of 3 to 6 years old. Therefore, this developmental period is held up as a key stage in acquiring those skills identified as predictors of school performance. These precursors can be differentiated into two categories: domain-general and domain-specific (Passolunghi & Lanfranchi, 2012). On the one hand, precursors of domain-general ability refer to general cognitive skills that predict performance in school topics, not in one particular field. Examples include short-term memory, working memory and general intelligence (Aragón, Navarro, Aguilar, & Cerda, 2015).

On the other hand, domain-specific precursors refer to those skills that are able to predict subsequent performance in a particular area of school knowledge. Such precursors include phonological awareness as a predictor of literacy skills, and skills related to number sense acquisition and counting which are critical in mathematical skills achievement (De Smedt, Verschaffel, & Ghesquiere, 2009).

Acquiring number sense is completed through a gradual process. Four different stages in the development of number sense can be identified (Von Aster & Shalev, 2007). First, the child achieves non-symbolic representation. Secondly, non-symbolic representation leads to symbolic representation of quantity. Thirdly, verbal symbolic representation is acquired through Arabic numerals, and finally, the previous three components are integrated into a mental number line, i.e., when it is assumed that numbers listed after in a counting sequence are bigger numbers than those listed before. Therefore, at the age of 6, after reaching the fourth stage of number sense development, most children incorporate

notions of quantity and counting schemes within mental number lines (Siegler & Booth, 2004).

This linear representation of numerical magnitude is significant to the development of counting skills, as it helps children learn the positional value of numbers and do mental calculations. Consequently, number sense remains as a strong predictor of achievement in math in early school years (Duncan et al 2007; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009).

On the opposite, problems in different developmental stages of number sense, identified by Von Aster & Shalev (2007), may explain the emergence of learning difficulties in mathematics (Mazzocco, Feigenson & Halberda, 2011; Van Viersen, Slot, Kroesbergen, Van't Noordende, & Leseman, 2013). Therefore, shortfalls in number sense can make formal math instruction difficult (Baroody & Rosu, 2006). It can be considered a determining factor in the onset of dyscalculia (Butterworth, 2010; Piazza et al, 2010), and explains the disparity in math performance in the early grades of primary education (Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2013). It is therefore critical to pay attention to the several aspects of numeric in the early school years, as well as the previous processes, which according to Piaget (Piaget & Szeminska, 1941) contribute to constructing the number concept. Therefore, to successfully meet the demands of formal schooling, it is necessary to acquire and coordinate beforehand, relational logic skills.

A key aim is to prevent mathematical learning difficulties (MLD) and teach early math skills. This goal can be achieved through training programs that contribute to the student's progress and help not only for academic purposes, but also provide strategies to meet the demands of the social world in which the student is immersed. There are a wide range of tools and teaching methods that can help achieve the proposed goals, including computer-assisted instruction (CAI). Research supports the role of CAI as a mediator tool in improving higher order capacities (Ayvaci & Devecioglu, 2010; Döst, Saglam, & Ugur Altay, 2011; Halpern, Millis, Graesser, Butler, Forsyth, & Cai, 2012), critical to achieving effective competency in several school subjects. For more than a decade, research has supported the value of this CAI teaching method in specific school skills such as mathematics (Clements & Sarama, 2007, 2008, 2011; Griffin, 2004; Sarama, Clements, Starkey, Klein, & Wakeley, 2008).

In order to study the effectiveness of an intervention program based on computer-assisted instruction, the following objectives were established: First, to analyze the differences between control and experimental group scores after the intervention process, in both relational and numeric mathematics tasks, and the total test score which assesses early math skills. A detailed study of several math subtests activities was also proposed. On the one hand, differences in relational tasks were analyzed: comparisons, classifications, correspondence and seriation tasks. On the other, the differences found in numeric tasks were studied: verbal counting, structured counting, resulting counting, general knowledge of numbers and estimation.

Method

An experimental design was used, with experimental and control groups, and pre- and post-intervention measures. The SPSS.22 version was used for data analysis. Through this software, descriptive statistics of the sample were calculated and an inferential analysis was carried out through the Mann Whitney *U* test for two independent samples and discriminant analysis. The researchers obtained the permits and ethical recommendations required for these type of studies. Parents, teachers and school board granted permission for the study. After the intervention, the school teachers that had participated in the study were guided to improve student performance, using the material implemented in the research in order to increase the skills of students.

Participants

The sample of students came from four schools located in the Cadiz province (Spain). Two schools were public, and two were semi-private. The schools had a standard middle class socio-economic level. There were a total of 48 students from the final grade of Early Childhood Education, whose ages ranged between 4.91 and 5.91 years, ($M = 5.4$; $SD = .29$). 27 participants were girls, aged between 4.91 and 5.83 months ($M = 5.37$, $SD = .27$) and 21 participants were boys, aged between 4.91 and 5.91 months ($M = 5.43$, $SD = .32$). Students who had special educational needs were excluded.

Instruments

The Early Numeracy Test-R (Van Luit and Van de Rijt, 2009) in its computerized and standardized version translated into Spanish (Van Luit et al., 2015) was used to evaluate the participating students' math skills. Based on the students results, they were selected to develop early math skills through «Playing with Numbers 2» (Navarro, Ruiz, Alcalde, Aguilar, & Marchena, 2007)

Playing with Numbers 2

The software «*Playing with Numbers 2*» (Navarro et al., 2007) is a training program aimed at learning, developing and strengthening mathematical thinking skills. Its aim is to contribute to the student's logical-mathematical acquisition in an attractive and motivating way, using the computer as a learning facilitator. Specifically, «Playing with Numbers 2» was carried out with Flash Professional Adobe software, which uses standard internet files (SWF) and vector graphics so that different scale applications can be used without losing visual quality.

The software consists of different tasks to develop number sense. This computerized training program offers students activities that help them learn concepts related to certain basic math skills such as: seriation, comparison, classification, easy arithmetic problems, distribution, size discrimination, and a series of activities aimed at mastering numbering. «Playing with numbers 2» is aimed at students of the first year of primary school. This software has different levels of difficulty and has the possibility of being used with younger students, or students with special educational needs.

Early Numeracy Test (ENT-R)

A computerized version of Early Numeracy Test Revised (ENT-R) was used in this research, standardized for the Spanish population (Van Luit et al., 2015). The original tool was developed by Van Luit and Van de Rijt (2009) to evaluate early numerical knowledge, and detect students at risk of MLD. This tool is especially useful in the transition from pre-school to

elementary education, in order to confirm which students need support to cope with the new mathematical learning, promoting the implementation of early intervention to remedy these shortfalls.

The test assesses concepts of comparison, classification, one to one correspondence, seriation, verbal counting, structured counting, counting (without pointing), general knowledge of numbers and estimation. It is aimed at students between 4 and 7 years, and has three parallel versions of 45 items each. It takes an average of around 30 minutes and is individually administrated. The Cronbach's alpha reliability index was .92.

Procedure

First, parental permission to participate in this research was obtained for the student sample selected. Then, an individual evaluation session was carried out at the beginning of the school year. In this first assessment phase, the ENT-R (version A) test was applied to assess early math skills. The individual evaluation session took between 30 and 35 minutes.

The second phase of the study focused on the implementation of the computerized intervention program «Playing with Numbers 2». The training program was carried out during the months of February, March and April. A total of 30 sessions were used, taking between 30 and 45 minutes each. They were held three times a week. Each training session involved 6 students working independently with a computer, solving the different computer activities. All sessions were supervised by two specialized psychologists.

Finally, we proceeded to perform the post-test evaluation by the ENT-R (version B) in order to verify the effectiveness of the intervention.

Results

The Mann Whitney *U* nonparametric test was used on independent samples before and after the intervention, in order to verify the existence of significant differences between the control and experimental groups before and after the intervention.

TABLE I. Results of Mann Whitney U test for two independent samples for complete test, relational and numerical subtests and task groups.

	Pretest	Posttest
Complete Test	.115	.001*
Relational subtest	.607	.001*
<i>Comparison</i>	.864	.447
<i>Classification</i>	.407	.001*
<i>Correspondence</i>	.623	.001*
<i>Seriation</i>	.039	.073
Numerical subtest	.423	.003*
<i>Verbal counting</i>	.435	.016
<i>Structured counting</i>	.011	.001*
<i>Resultant counting</i>	.016	.001*
<i>General Knowledge of numbers</i>	.197	.074
<i>Estimation</i>	.783	.143

* $p < .001$

Table I shows the results obtained from the Mann Whitney *U* *contrasting test*. In the pretest assessment, no significant differences were found in the complete test, nor were they found in the subtests or the different components of early numerical knowledge. There was statistical equivalence between the control and experimental groups. In the posttest, after the intervention, significant differences in ENT-R total test score and relational and numerical subtests were found. With regard to the diverse subtest components, significant differences between control and experimental group were found in some of the elements evaluated. On the one hand, with respect to the relational subtest, there were significant differences in classifications and correspondence ($p < .001$) but differences in comparison and seriation were not significant. Furthermore, the differences were statistically significant in structured counting ($p < .001$) and resulting counting ($p < .001$). However no statistically significant differences were found in verbal counting or the general knowledge of numbers and estimation.

A detailed descriptive statistics analysis generated some more significant information (see Table II). The descriptive results regarding the score obtained in all subtests and the increase achieved after the

intervention program, are shown in Table III. Table IV shows descriptive statistical results obtained on the different components of early numerical knowledge.

TABLE II. Means and Standard Deviations obtained in the total score on the Early Numeracy Test-R by experimental and control groups, and the increase achieved after the intervention program.

	N	ENT-R total score				Increase
		Pretest		Posttest		
		M	SD	M	SD	
Experimental group	24	15.88	1.45	28.04	4.95	12.16
Control group	24	16.54	1.50	22.00	3.77	5.46
Total	48	16.21	1.50	25.02	5.32	8.81

Results in Table II confirmed that despite the experimental group being initially higher in the pretest than the control group (although these differences between groups were not significant), the control group performed less well than the experimental group after the intervention program.

In addition, gains obtained by the experimental group were 6.7 points significantly higher than those in the control group.

TABLE III. Results obtained in relational and numerical subtests of the Early Numeracy Test-R by experimental and control groups, and the increase achieved after the intervention program.

	N	Relational subtest					Numerical subtest				
		Pretest		Posttest		Increase	Pretest		Posttest		Increase
		M	SD	M	SD		M	SD	M	SD	
Experimental group	24	10.13	2.25	14.54	1.95	4.41	5.75	2.15	13.50	3.86	7.75
Control group	24	10.33	1.60	11.38	2.24	1.05	6.21	2.00	10.63	2.49	4.42
Total	48	10.23	1.93	12.96	2.62	2.73	5.98	2.06	12.06	3.53	6.08

On the other hand, Table III shows the absence of pretest significant differences between experimental and control groups (see Table I).

Pretest scores were comparable and both groups considered equivalent. After the program implementation, the experimental group score emerged as higher than the control group. Differences were statistically significant. With regard to the increase of both groups, the experimental group had a three points higher advantage than the control group. These results endorsed the effectiveness of intervention beyond developmental changes and school instruction.

Table IV shows results obtained in the different critical components involved in comprising the ENT-R. The increase was higher in all tasks for the experimental group compared to the control group. These differences supported the role of the computerized intervention. As shown in Table I, the differences were statistically significant in correspondence and relational classification tasks. Similarly, the increasing was greater in the experimental group. The size effect was also more significant for the experimental group (Table IV).

TABLE IV. Results obtained for experimental and control groups on the ENT-R different tasks.

	Experimental group						Control group					
	Pretest		Posttest		f*	d	Pretest		Posttest		f*	d
	M	SD	M	SD			M	SD	M	SD		
Comparison	4.38	.711	4.83	.381	.45	.78	4.33	.917	4.71	.550	.38	.50
Classification	1.50	.834	2.75	.897	1.25	1.44	1.29	.859	1.75	.847	.46	.53
Correspondence	2.63	1.13	4.25	.737	1.62	1.69	2.46	1.02	2.71	1.08	.25	.23
Seriation	1.63	1.24	2.71	1.12	1.08	.91	2.25	1.07	2.21	1.06	-.04	-.03
Verbal counting	1.17	.963	2.96	.955	1.79	1.86	1.00	1.10	2.29	.955	1.29	1.25
Structured counting	1.04	.806	3.46	1.06	2.42	2.57	1.63	.824	2.38	1.05	.75	.79
Resulting counting	1.50	.659	3.00	1.06	1.50	1.69	.92	.830	2.00	.659	1.08	1.44
General knowledge of numbers	1.21	1.28	3.13	.90	1.92	1.73	1.67	1.239	2.58	1.06	.91	.79
Estimation	.83	.917	1.38	1.27	.55	.49	1.00	1.18	1.38	1.27	.38	.30

Next, the impact of the effect on both groups was measured. Aside from normal maturing processes, the control group increased its score as a result of traditional school curriculum instruction whilst the experimental group improved from both types of instruction, i.e., traditional curriculum and computer instruction using the software

«Playing with numbers 2». According to Table IV the impact on classification skills was higher in the experimental group ($d = 1.44$), compared to the control group ($d = .53$). In addition, the impact in the correspondence task was also higher in the experimental group ($d = 1.69$) than in the control group ($d = .23$).

On the other hand, with respect to results in ENT-R numerical subtests, the experimental group displayed a greater impact ($d = 2.57$) than the control group ($d = .79$) in structured counting task. Similarly, the experimental group ($d = 1.69$) also displayed a greater effect than the control group ($d = 1.44$) in the resulting counting task.

With regard to the increase achieved on the posttests, it is noteworthy that both the experimental and control groups had higher scores in numerical tasks than in relational tasks, except in estimation where gains were not as high as in the other types of numerical tasks.

Finally a discriminant analysis was conducted in order to establish differences between groups and obtain a mathematical function able to classify students based on the scores obtained in the discriminating variables. This statistical technique allows a supervised classification of data vector (numerical) into two or more categories: in this case, control or experimental group. This classification is based on an achieved hyper plane able to distinguish the experimental and control groups. This distribution was compared with the real results giving a classification matrix where the diagonal represented the total or percentage of well classified individuals and where extradiagonal elements represented the false positive and false negative classification process (Table V).

TABLE V. Results of discriminant analysis to predict students belonging to the control and experimental group.

		Predicted group adjustment		Total
		Experimental group	Control group	
Re-counting	Experimental group	20	4	24
	Control group	4	20	24
%	Experimental group	83.3	16.7	100
	Control group	16.7	83.3	100

The results in Table V show that 83.3% of students in the control group were correctly classified in their group. The same percentage was also

obtained for the experimental group. As a result, it was possible to conclude that the intervention program produced a statistically robust differentiation between both groups.

Additionally, an equal groups statistical contrast based on the Wilks' *Lambda*, and determined by a *Chi-square* approximation was calculated. Results rejected the hypothesis of equality between the groups (*Wilks' Lambda* = .468; $X^2 = 31.46$; $p < .0001$). The conclusion was that differences between experimental and control groups were statistically significant.

Discussion

This paper provides a complementary tool to traditional math instruction, efficient in developing early math skills using new technologies. It also studied which cognitive skills were improved by the intervention program.

According to the results, blending traditional instruction with new ways of teaching mathematics at an early age contributes to number sense acquisition. Number sense is defined as a set of related knowledge of number and operational skills. These skills are specific ability precursors of mathematical performance in subsequent years and are a key element in Mathematics Learning Disabilities (Jordan, Glutting, & Ramineni, 2008; Jordan et al, 2009; Passolunghi et al., 2012). One way to prevent these potential difficulties and contribute to improving early skills is to provide tasks to enhance number sense as the milestone of mathematical learning in early years (Aunio, Hautamäki, & Van Luit, 2005; Geary, 2004).

According to the Piagetian model (Piaget & Szeminska, 1941), logic-relational early math skills are meaningful to achieve number understanding. Therefore, insisting on providing activities based on serialization, correspondence, comparison and classification, contributes to developing numerical knowledge and, eventually, to an encouraging prognosis of students' math skills in higher school grades (Aunio & Niemivirta, 2010). This is especially remarkable in exploring new instructional methods such as computer assisted teaching. Advantages derived from research suggest that this procedure has the ability to adapt contents to the students' learning needs, individually adjusting and personalizing curriculum (Judge, Puckett, & Cabuk, 2014).

With respect to the results after the intervention applied in this study, students in the experimental group showed significant differences from the control group in relational, classification and correspondence tasks. However, no differences were found in comparison and seriation tasks. It is possible that students had already mastered this ability through developmental maturation at the time this was first tested. Actually after intervention program, increases in this task score were just 0.5 point between both groups.

The rationale for this lack of difference should be based on results of an exploratory study to adapt the ENT-R test (Araújo, Aragón, Aguilar, Navarro, & Ruiz, 2014). Comparison tasks in 5 year old children were those that yielded more positive results, compared to all other tasks. Furthermore, items corresponding to comparisons were less difficult for students, as we found in the current study. From a theoretical point of view and after the statements by Resnick (1989, 1992), the proto-quantitative comparison scheme would conduct the first steps of quantity, with no numerical precision. Students should assign linguistic labels without any measurement process. It is therefore known that these comparison skills are acquired before age 5, the stage at which it is in the consolidation process.

On the other hand, with respect to seriation concepts, it is known that together with classification, both are progressively coordinated to master numbers as serial categories. Number conservation is intrinsically linked to the process of integrating these logical abilities (Piaget & Szeminska, 1941). For that reason, its acquisition is essential for the development of early math skills. As Fuson (1991) pointed out, seriation skills are reached at the last stage of numerical sequence development. It is characterized by being two-way and staggered. That is why most students despite having worked more or less on this ability, are not able to master this skill that -along with the inclusion of classes-, represents the last basic maturational step in mastering the numerical sequence. This idea is also supported by Fernandez and Ortiz (2008) who identify several stages for acquiring the logical ordinal-number sequence and whose improvement is achieved after the age of 5.

In addition to the development of logical-relational thinking, it is essential to learn the conventional numbering system derived from contextualized and meaningful school learning (Berch, 2005; Bryant & Nunes, 2002). This learning also predicts subsequent achievement in

student performance (Gersten et al., 2005; Aunio & Niemvirta, 2010). Therefore, insisting on complementary instructional school work with tasks that polish the skills acquired in the classroom can be beneficial for student performance in math.

The results of the intervention program in relation to the numerical tasks showed significant differences for the experimental group in resulting, structured and counting tasks. However, there were not significant differences in verbal counting or general knowledge of numbers and estimation. Five year old children are easily able to properly recite the number sequence. But it is more difficult to solve counting (and without pointing) tasks. These tasks could be improved with specific training. On the other hand, general knowledge of the number tasks requires a certain generalization of everyday life mathematical basic knowledge. This is regularly more complex to reach at an early age. Nevertheless, the intervention program demonstrated that participants in the experimental group with lower scores in the pretest assessment, achieved higher scores than the control group in general knowledge of number tasks, without reaching statistically significant differences.

Finally, with respect to estimation tasks, it is necessary to consider that providing meaning to the numbers' magnitude on a number-line continues to be one of the most complex mathematical components for students at five years of age (Araújo et al., 2014). Several studies (Booth & Siegler, 2008; Siegler & Booth, 2004) have focused on estimation using a number-line for nursery and primary education students. Results found that estimation errors significantly decreased when children finished pre-primary education and were also much more accurate when students estimated numbers that correspond to a hundred rather than a thousand (Siegler & Opfer, 2003). All these results suggested that despite the training program, it is necessary for students to develop further basic math concepts such as verbal-number string, which supports the subsequent acquisition of a more complex knowledge, such as estimation.

From a more overall approach, we find higher gains in relational tasks than numerical tasks in the experimental group. Also, trained students obtained more than three points in both relational and numerical aspects. According to Resnick (1989, 1992), self-intuitive knowledge of logical tasks represents a cornerstone in future math skills acquisition. Consequently, it is necessary that this non-instructional and relational previous knowledge, be integrated with the representational knowledge

led by counting, in order to cope with cognitive conflicts as they come up. It is therefore rational that at 5 years of age training gains are focused on strengthening these intuitive features and increasing the development of skills required through instruction and active learning.

Regarding methodological limitations in the study, we can mention that the study shows characteristic limitations of research in education. In this case, we can highlight issues related to randomness in the sample, and therefore, the realization of a type of quasi-experimental design, for example, the ecological variables in schools, such as the influence of the characteristics of teachers in the teaching of mathematics, along with the reduced number of participants in the study due to the need of resources that must be taken into account for intervention.

In conclusion, the implementation of a mathematics training program in early years based on computer assisted instruction was helpful to enhance complementary math skills to school teaching. Results supported the usefulness of training considering child developmental and math instruction received at school. The use of the software «Playing with numbers 2» by the experimental group as extra training to traditional instruction, was efficient for early math learning skills.

The follow up of all participants and the analysis of its progress is a new target for future research as is how to maintain the impact and how to respond to the intervention in subsequent school years. Likewise, there is the option of introducing the software «Playing with numbers 2» in classrooms integrated as a standard tool, together with other traditional math teaching approaches, enhancing benefits of computerized intervention to all students without disrupting the class dynamic.

On the other hand, with respect to the piece of software, it is necessary to review tasks with a potential ceiling effect at these ages, and whether or not to introduce or modify tasks, which are not beneficial due to their complexity.

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