INVESTIGATING STRATIFICATION, LANGUAGE DIVERSITY AND MATHEMATICS CLASSROOM INTERACTION

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Research on the socio-political dimensions of language diversity in mathematics classrooms is under-theorised and largely focuses on language choice. These dimensions are, however, likely to influence mathematics classroom interaction in many other ways than participants' choice of language. To investigate these influences, I propose that the notions ofheteroglossia, orders of indexicality and scale-jumping, can provide new theoretical tools with which to understand the links between classroom interaction and broader social patterns of marginalisation. To illustrate the utility of these ideas, I include some analysis of an episode observed in a sheltered elementary school second language mathematics classroom in Canada.

Keywords: Bilingual learners; Language diversity; Mathematics education; Socio-political dimensions; Stratification

Investigando la estratificación, la diversidad lingüística y la interacción en el aula de matemáticas

La investigación sobre las dimensiones sociopolíticas de la diversidad lingüística en clases de matemáticas está poco teorizada y mayormente se centra en la elección de la lengua. Estas dimensiones, no obstante, probablemente influyen en la interacción en clase en otros modos distintos a la elección de la lengua. Para investigar estas influencias, propongo que las nociones de heteroglosia, órdenes de indexicalidad y salto de escala, pueden aportar nuevos instrumentos teóricos con los cuales comprender conexiones entre interacción del aula y patrones sociales de marginalización. Para mostrar la utilidad de estas ideas, incluyo los análisis de un episodio de una clase de primaria de matemáticas canadiense con instrucción en una segunda lengua.

Términos clave: Aprendices bilingües; Dimensiones socio-políticas; Diversidad lingüística; Educación Matemática; Estratificación

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The impact of language diversity in mathematics classrooms has been the focus of an increasing amount of research (see, for example, Barwell, 2009; Barwell et al., 2016; Halai & Clarkson, 2016). Language diversity in mathematics classrooms can take many forms. For this paper, I use language diversity to refer to any classroom in which any of the participants uses more than one language in their daily life. This definition includes classrooms commonly referred to as bilingual or multilingual or second language classrooms, as well as classrooms in which the languages of some students are not used and not recognised. This definition is consistent with contemporary sociolinguistic perspectives that challenge rigid separations between languages or language situations (e.g., Blommaert, 2010; Makoni & Pennycook, 2007).

Research on language diversity in mathematics classrooms can be organised into three main groups: cognitive research, discursive research, and socio-political research (Barwell, 2014a). Cognitive research has mostly looked at language diversity from an individual perspective to understand how it influences their mathematical thinking and performance (e.g., Dawe, 1983). In this work, individuals who use multiple languages, such as bilingual learners, multilingual learners, or second language learners are the main unit of analysis. Discursive research looks at how language diversity affects students' participation in mathematics classroom interaction (e.g., Moschkovich, 2009). The main unit of analysis in this kind of research is classroom talk, rather than individual learners or teachers. Finally, socio-political research has sought to highlight how language is not simply a means of communication or a tool for thought; the way language is used means that some participants may be privileged in different ways, while others may be marginalised (e.g., Setati, 2008). Such influences are sometimes systemic and reflect wider social forces, such as those of racism or class. Of course, these groupings are quite general and are not mutually exclusive. Much socio-political research, for example, deploys discourse theories and methods. The characterisation is simply a way to make sense of underlying trends in the research literature.

In this paper, I argue that research in the socio-political group remains theoretically under-developed. I develop this argument in the next section, in which I look at some of the literature in more depth. I then seek to contribute to the necessary development of this area of research by drawing on theoretical ideas from the contemporary sociolinguistics of multilingualism, include the concept of orders of indexicality. These theoretical ideas make it possible to link general notions of the political role of language in mathematics classrooms to the subtleties of everyday mathematics classroom interaction. I illustrate these ideas with a brief example of data analysis from an ethnography of second language mathematics classrooms in Canada. My aim is to demonstrate that these theoretical ideas make it possible to develop a more nuanced understanding of how language is implicated in the stratification of students' participation in mathematics and hence how it has an impact on their opportunities to learn mathematics.

Socio-Political Research on Language Diversity in Mathematics Education

Research on mathematics classroom interaction in multilingual settings dates back at least to the 1990s. Much of this work has adopted a view of language as a "resource". Research on teaching practices includes Adler's (2001) identification of dilemmas that arose for several teachers in different multilingual mathematics classrooms in South Africa, Khisty's (1995) comparison of three teachers in Spanish-English bilingual classrooms in the USA, and Moschkovich's (1999) study of a Spanish-English bilingual mathematic class also in the USA. These studies highlight the challenges many teachers face in working with students who draw on multiple languages in the mathematics classroom. The dilemma (to use Adler's term) of whether or not to give explicit attention to mathematical language or focus on the mathematical ideas was a challenge that emerged in all three studies. Research on students' participation, meanwhile has identified several resources on which students may draw in mathematical discussion. These resources include code-switching (Planas & Setati, 2009; Setati, 2005); genre and narrative (Barwell, 2003); and gestures, writing and diagrams (Moschkovich, 2009). While the majority of these studies show some awareness of the socio-political dimension of language, this awareness is not always apparent in the design and conceptualisation of the research.

In recent years, research has emerged that gives more explicit attention to the socio-political dimension (e.g., Norén, 2015; Planas, 2011; Planas & Civil, 2013; Setati, 2005, 2008). Much of this research is actually based on discursive analyses. For example, Setati's (2005, 2008) work is based on Gee's concept of cultural models (Gee, 1999), which he uses to refer to shared assumptions about how the world works.

In one study, for example, Setati (2008) interviewed mathematics learners and teachers in multilingual South Africa, in which, in principle, mathematics can be taught in any of 11 official languages. Setati used the concept of cultural models "to explore why teachers and learners prefer the language(s) that they choose for learning and teaching mathematics" (p. 105). In her analysis, Setati uncovered a cultural model about English as an international language that provides access to jobs and higher education. This cultural model appeared to dominate decisions about which language should be used to learn and teach mathematics, even when participants expressed some awareness that this choice made learning mathematics more challenging. This study clearly highlights a socio-political tension about the choice of language for learning and teaching mathematics in a context of language diversity. The cultural model of English as an international language is political in nature—it privileges a particular language and people who are able to use it effectively. Moreover, learning English is seen as more important than learning mathematics; given the choice, learners preferred to learn English at the cost of a deeper understanding of mathematics. While this study, and several others by Setati, serves to highlight the political role of language in the context of language diversity in mathematics classrooms, it does little to theorise the nature of this role and, in particular, its impact on classroom interaction.

A more theoretically robust approach has recently been proposed by Planas and Setati-Phakeng (2014) based on a framework from language planning research. The framework, first developed by Ruiz (1984), distinguishes three approaches: language-as-right, language-as-resource, and language-as-problem. Planas and Setati-Phakeng reject the language-as-problem approach and seem to support the language-as-right approach. Indeed, in the two contexts in which they work (Catalonia in Spain and South Africa), language rights are prominent in political debate. They argue, however, that language-as-resource is an approach that is closer to the reality of daily classroom life and it also validates much of the previous research I have already referred to, which identifies various different language resources used by mathematics learners in contexts of language diversity.

The notion of language as resource is, however, in pedagogical terms, not straightforward. Planas and Civil (2013) investigated the language choices of students and teachers in mathematics classrooms in Catalonia and Arizona. Rather like Setati (2008), they found that the pedagogical value of students' home languages may be overridden by broader political considerations. As a result of their analyses, they concluded that the useof language as a resource by mathematics teachers is mediated by political considerations. The resulting tension boils down to the long-standing issue in second language settings of whether to focus more on ensuring students learn the language of instruction, or whether to focus more on ensuring that they learn mathematics. This is, in fact, a recurrence of the dilemma mediation noticed by Adler (2001) and prevalent in many other studies (e.g., Barwell, 2012).

It is fairly clear, then, that the socio-political dimension of language influences what happens in mathematics classroom interaction. The research I have discussed has tended to focus, however, on one single aspect of language use: the choice (if it is a choice) of language. Thus, in Setati's (2008) work, the choice is between English and an African language or languages. In Planas and Civil's (2013) study, the choice is between Spanish and English, or Catalan and Spanish. The socio-political dimension of language is, however, likely to influence mathematics classroom interaction in many other ways than participants' choice of language. I do not mean to imply that the choice (or non-choice) of which language(s) may be used in a mathematics classroom is not important, or not political in nature; rather, I suggest that the socio-political dimension of language is likely to influence many other aspects of language use in mathematics classrooms, but these aspects have received much less attention. A stronger theorisation of the socio-political dimensions of mathematics classroom interaction therefore needs to be broader in scope than a focus on language choice only.

Heteroglossia and Orders of Indexicality

There have been some significant shifts in how multilingualism (and language itself) is conceptualised and understood in recent years. Many of these shifts can be traced, in part, to the work of Bakhtin (1981) who developed a view of language as situated, dialogic, and tension-filled. Bakhtin's ideas have led to a view of multilingualism that, rather than focusing on discrete, clearly defined languages and associated clearly defined groups of speakers, looks at language as social practice situated in social and political contexts (Blackledge & Creese, 2010, p. 25). More specifically, Bakhtin (1981) defines the key concept of heteroglossia as "the social diversity of speech types" (p. 263). He describes this diversity as follows.

> *At any given moment of its evolution, language is stratified not only into linguistic dialects in the strict sense of the word [...] but also [...] into languages that are socio-ideological: languages of social groups, "professional" and "generic" languages, languages of generations and so forth.* (Bakhtin, 1981, pp. 271-272)

Heteroglossia, then, refers to the many patterns that arise within language and which can be associated with some group of people, situation, activity or other social formation. Bakhtin (1981) describes these distinctions in terms of the stratification of language. There are a couple of important points to note about this account. First, the many different patterns within the diversity of language overlap and intersect. The language of teachers, the language of mathematics and the language of a particular region may all be present in the same utterance. Moreover, the distinctions between the speech types to which Bakhtin (1981) refers are produced by these practices; they are not pre-given. Thus, what counts as an accent, as "teacher talk" or even as a language, is locally produced (Bailey, 2007). The way that language practices can "point to" such associations, allowing us to recognise particular activities, group memberships or situations, is called *indexicality*. This aspect of language is important in framing particular utterances, so making them interpretable.

Bakhtin (1981) proposes that heteroglossia is one "pole"; the other he refers to as "unitary language". This pole represents the idea that languages are rule-governed systems. If heteroglossia represents the idea that humans shape language, "unitary language" represents the idea that humans must follow the pre-existing rules of language. No such rules exist, of course, but the idea that they do is widespread in popular culture, apparent in the importance attached to correct spelling, "good" diction and the existence of bodies like the *Académie Française* whose function is to patrol the linguistic frontiers and ensure that the language stays pure. Unitary language is, moreover, closely linked with social stratification: So-called "standard" forms of using language are typically the forms of the governing classes. This concept explains, for example, the preference for English for learning mathematics observed in Setati's (2008) study. In South Africa, English is the force of unitary language and is the language of the elite, successful and powerful. Indeed, the tensions between heteroglossia and unitary language can be seen in much of the research on language diversity in mathematics education around the world (see Barwell, 2012, 2014b).

Second, the tension between heteroglossia and unitary language and indexicality, make a link between specific instances of language use and broader social patterns and forces, like Blackledge and Creese (2009) suggest.

> *Linguists have increasingly turned to the works of Bakhtin and his collaborator Volosinov because their theories of language enable connections to be made between the voices of social actors in their everyday, here-and-now lives and the political, historical, and ideological contexts they inhabit. In familiar terms, Bakhtin's philosophy of language contributes to the means by which we may understand the structural in the agentic and the agentic in the structural; the ideological in the interactional and the interactional in the ideological; the "micro" in the "macro" and the "macro" in the "micro".* (pp. 237-238)

Ways of talking both reflect the socio-historical dimension of language and create this dimension for the future. So when learners prefer English for learning mathematics in South Africa, they are influenced by the perceived role of English as the language of the elite, successful and powerful. At the same time, they are also reproducing this stratification each time the make a choice in favour of English. The link between micro and macro highlighted by Blackledge and Creese offers valuable traction in linking classroom interaction with the broader socio-political dimensions of language use in mathematics classrooms. Individual utterances (micro) shape and are shaped by broader socio-political language forces (macro).

Third, these different ways of talking are stratified; some ways of talking are considered more valuable than others, as already discussed in relation to the concept of unitary language. However, unitary language is a fairly crude account of stratification. Blommaert (2010) offers a more nuanced account that can more easily be applied to the analysis of mathematics classroom interaction than the crude account of stratification implied by the concept of unitary language. This account is based on the concept of indexicality, which I explain in what follows.

> *Ordered indexicalities operate within large stratified complexes in which some forms of semiosis are systemically perceived as valuable, others as less valuable and some are not taken into account at all, while all are subject to rules of access and regulations as to circulation. That means that such systemic patterns of indexicality are also systemic patterns of authority, of control and evaluation, and hence of inclusion and exclusion by real or perceived others.* (Blommaert, 2010,p. 38)

The stratification of language can therefore be seen to be linked to inclusion or marginalisation through the way different ways of talking or writing are linked to more or less valuable or powerful groups or activities. This kind of stratification typically maps onto scalar differences in practices, so that local (i.e., present on a small scale), idiosyncratic practices are perceived as less valuable than more widely used (i.e., present on a wide scale), standardised practices (Blommaert, 2010, p. 35). Blommaert (2010) discusses several examples to illustrate the ideas of ordered indexicalities and scalar differences. For one, he refers to a price list for cold drinks found in London's Chinatown (p. 31). The price list is written in Chinese characters and in English. The English includes "quite spectacular typos" (p. 31), such as "Lced" for "Iced" and "Coffce" for "Coffee". Blommaert (2010) points out that for many customers in London, the Chinese characters are "a meaningless design", but which index "Chineseness" and a link with wider Chinatown. He also imagines the sign being printed somewhere in China, where the English would be equally meaningless, simply symbols to be reproduced in printed form. To customers in London, the spelling mistakes might be a source of amusement, but might also index less favoured or less valuable forms of English literacy. Hence, indexicality is itself situated, dependent on who is producing or interpreting language or text, as well as where they are and what they are doing. Blommaert (2010) illustrates this point stating that "the English spoken by a middle-class person in Nairobi may not be (and is unlikely to be) perceived as a middle-class attribute in London or New York" (p. 38).

The idea that language use indexes social stratification provides the link between micro moments of interaction and broader social patterns and forces. This account has the potential to make possible analyses of mathematics classroom interaction to explain some of the socio-political effects previously identified in the literature. For example, when a teacher in the study by Planas and Setati-Phakeng (2014) offers a translation of a word in a mathematics problem to help a student who is learning Catalan, she is managing the heteroglossia of her classroom in a way that indexes both the less preferred language of the student and the more "valuable" standard ways of talking about mathematics in Catalan. It is through this indexicality that stratification is produced, particularly if patterns of indexicality develop over time: In mathematics classrooms, it is commonly the student's ways of talking about mathematics that are positioned as less desirable and that are hence marginalised.

Blommaert (2010) calls these indexical shifts (from a less preferred to a more preferred form of language) "scale jumping" since such shifts often reflect effects of scale. In the moment in which the teacher in Planas and Setati-Phakeng's (2014) study translates a word in the problem for her student, she is invoking a way of talking that is the more widely used. The student's use of Spanish in many contexts would reflect the more widespread and dominant language; in Catalonia, however, this is not the case.

In the next section, I illustrate the potential of these theoretical ideas for more carefully analysing and understanding how socio-political forces play out in the minutiae of mathematics classroom interaction.

Research Setting: A Second Language Mathematics Class

From 2008 to 2012, I conducted an ethnographic study of mathematics learning in different second language settings in Canada, a country with two official languages, English and French. The example I have selected for this paper comes from data from one of these settings, located in an Anglophone school in the French-majority province of Quebec. The data come from interactions recorded in a Grade 5-6 (aged 10-12 year) sheltered class for students identified by the school as English as a second language (ESL) learners (based on a standardised test of English reading comprehension), and as falling behind in both English and mathematics. The small group of students meeting these criteria therefore studied these two subjects each morning in a separate class from their regular classmates. I visited the class regularly throughout the 2009-2010 academic year. During that time, enrolment in the class varied quite a bit but never went over nine students.

For most of the year, all of the students in the class were Cree, one of the original peoples of Canada. The children's families were from the many Cree communities in the James Bay region of northern Quebec. The students spoke Cree as a first language. They also spoke English, though with a range of proficiency levels. In the move from James Bay to the city, the students went from being part of the majority in small Cree communities in which their language is widely used, to part of a minority in a city dominated by French and English. The language of the school and of their mathematics classes was English and their mathematics teacher was a monolingual speaker of English. Nevertheless, in some situations I did observe students speaking Cree with one another.

During my visits to the class, I acted as a participant observer, making field notes during teacher-led activities, and interacting with the students during small-group work. The teacher often asked me to work with small groups of students. I made numerous audio recordings of whole-class interaction and some small-group work, including my own work with groups of students. I collected samples of students' work and photographs of other artefacts, such as posters or work written on the blackboard. After each visit, I wrote a brief report summarising my observations.

For the purposes of illustration, in this paper I refer to an episode in which I worked with two students, Curtis and Ben, on a printed mathematical word problem about a tulip festival. The text of the problem was quite elaborate (i.e., more than a one-sentence word problem—see below) and I worked with the two students for approximately 20 minutes as they interpreted and solved the problem and wrote up their solution. I audio-recorded the entire episode and also took images of the students' completed written work. I have described aspects of this episode elsewhere (in particular, see Barwell, 2014b) but have not previously examined the stratification that arose in the students' interaction while they worked with each other and with me on the problem.

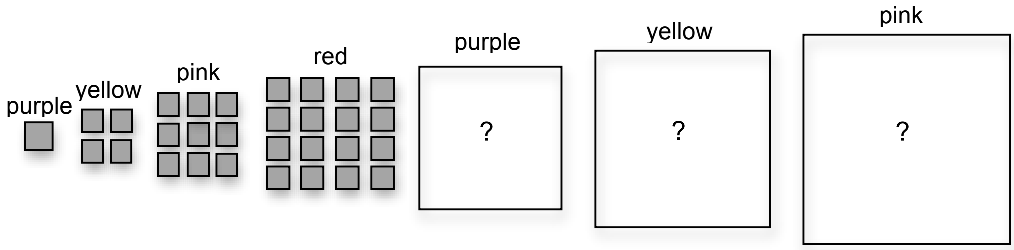
The Tulip Festival Problem

The word problem was presented in the form of a four-page booklet. The front cover included the title "Canada: Tulip Festival". The formulation of this problem is presented below.

> *Every year Ottawa holds a world-renowned tulip festival in the month of May. There are different gardens in various locations, one of which is on Parliament Hill. The Canadian Tulip Festival was established to honour Queen Juliana of the Netherlands, in 1953. It is the largest tulip festival in the world, making this flower the International symbol of friendship and the beauty of spring. This festival receives thousands of tourists every year from North America, Europe, and Asia and has an economic impact of approximately $50 million on the Ottawa region.*

The text continues on the next page, which I present in what follows.

> *You are a gardener hired to plant tulip bulbs for the Canadian Tulip Festival in May. You decided to arrange the flowers in a V for Victory format. You decide to use a pattern to make your design. Here is the design you started.*



How many purple, yellow, and pink tulips do you need to complete the design? Show all your work[1].

The problem text has a number of indexical features relevant to the students' subsequent work (and certainly this list is not exhaustive).

- The text is in English, which in itself indexes institutional norms (English as the language of the school board), provincial expectations (English as one of two official languages for education in Quebec), and English-speakers as the dominant language group in Canada.
- The introductory text giving the background to the tulip festival indexes factual registers typical of textbooks or informational texts, such as tourist brochures.
- The text indexes a particular place, Ottawa, and a particular event, the tulip festival, with which people in the region might be expected to have some familiarity. More specifically, and crucially for this episode, the text assumes a familiarity with tulips, a flower that is very common in the spring in this region. These associations, combined with the register used in the first part of the problem (e.g., Ottawa as the national capital, "world-renowned"), index a form of "Canadian-ness" reinforced by the reference to Canada's parliament.
- The full text is an elaborate form of word problem, with a scenario, some information and a mathematical calculation to be carried out. The presentation and structure of the text thus indexes the specific genre of mathematical word problems. This genre is widespread in Canadian mathematics classrooms (and in many other countries) and this particular form, known in Quebec as a situational problem, is a common form of assessment item in the province, used both locally as well as in the obligatory provincial mathematics tests that all students must take at the end of Grades 2, 4 and 6.
- The statement of the problem indexes the activity of gardening and the role of gardener ("you are a gardener"), perhaps as a way to link from the information about the tulip festival to the requirements of the mathematical task. The text makes assumptions about the role of a gardener, including the idea that a gardener can "decide" how to arrange the tulips (although, of course, the students cannot decide—their "deciding" has already been decided!).
- The text indexes certain mathematical forms (e.g., through the diagrams, vocabulary etc.), particularly geometric patterns and associated numerical

[1]The diagram is my reproduction of the slightly more elaborate version given to the students.

sequences indicated by the diagram, which is set out in a clear left-right progression.

Thus the text indexes a nation, a region, an event, speakers of a language, a register, a genre, and, finally, some mathematics.

To begin work on the problem, I asked Curtis and Ben to read the problem to themselves and then initiated a discussion about the content[2].

RB: Okay (.) so what's it about?

Curtis: Its about (.) world's biggest flower=I don't know.

RB: Ottawa's biggest.

Curtis: Tu (.) lip festival.

RB: Tulip festival (.) do you know any of those? (.) Do you know what a tulip is? [hm

Curtis: [Flower

RB: Flower right (.) have you ever seen a tulip?

[…]

Ben: (…) It's white.

RB: They are lots of different colours white ones red ones.

Curtis: Like a rose?

RB: Yellow ones say again.

Curtis: Rose

RB: No it's a bit different from a rose (.) roses yeah (.) tulips just come up in the spring and have a nice flower for about two weeks (.) then they are finished (.) there we go (.) let me see your picture.

Ben, Curtis: [Laughter]

RB: Have you seen flowers like that.

Ben: ^No^

Curtis: Yeah (.) in a store.

It is apparent from this exchange that the two students have some trouble interpreting the introductory informational text in the way it was presumably intended. For them, "tulip" initially indexes something rather vague: a kind of flower—hey mention roses and, at another point, poppies. And while the text might be designed to index a place and an event, and by extension, some aspects of Canadian-ness, the two students do not make this connection. In this way, the

[2] Transcript conventions: Short pauses are shown by (.), overlaps are shown by [, rising intonation is shown by?, emphasis is shown by bold type, and whispered speech is enclosed by ^ ^.

text serves to alienate the students, since although they are aware that tulips are a relevant point in the text, the text indexes ideas and perspectives that they do not themselves share. In other words, the text operates on a provincial and national scale (through choice of language, cultural references, etc.) while Curtis and Ben's reference points are more local. They are thus positioned marginal to a mainstream view of Canada.

Our discussion, which continues in similar vein to clarify what "bulbs" are and what "a gardener" does, can be read as an encounter between different "speech types" (Bakhtin, 1981): those of the text, the students, and me. Over the next few minutes, the students work at the problem, interacting with the diagram. I recorded the following observations in a note prepared after that day's visit to Curtis and Ben's class.

> *Ben moved first, drawing in rows of tulip bulbs in the boxes shown in the diagram. He did 5x5 in the first empty box and then moved on to the next box. Curtis looked at what he was doing and then did something similar. At some point, Curtis came up with a solution, fairly quickly. He just wrote three numbers at the bottom of the answer box. I didn't understand his solution but explained that he needed to explain how he worked it out. He wrote a sentence along the lines of "I added the tulips'—something quite general. So I said he needed to be more precise, to explain what calculation he did. At this point he explained to me verbally and I invited him to write it down. What struck me was that he had little trouble solving the problem, and that most of the time was spent on writing it down in an "acceptable" way.*

The fact that the two students are able to generate a solution to the mathematical problem represented by the diagram and the final problem statement suggests that they do relate to the mathematical pattern indexed by the diagram and are able to interact with it and, in particular, to extend it. At some level, then, there is some alignment in the forms of language (including graphic elements) used in this part of the text, and the students own linguistic repertoires. The way in which they express their solution and their reasoning, however, remains "local" (three numbers at the bottom of the answer box); that is, it makes sense to them but does not index more widespread forms of mathematical discourse. My notes record my observation that what the students found more challenging was writing their solution in an "acceptable" way. Of course, what is considered "acceptable" in this episode is largely determined by me based on my familiarity with mathematical discourse. That is, the students' efforts to produce an "acceptable" written solution are in part driven by my prompting. This point is also apparent in the audio-recording, including the following explicit formulation.

RB: You have to explain now that you've got these totals okay otherwise if somebody comes along and reads it they will wonder where the number comes from in these kinds of situational problems its quite important that you explain somehow how you worked it out.

My instructions here indicate to the students that the goal (my interpretation of the goal of the task within its institutional context) is to write in a way that is interpretable to some kind of generalized "somebody" and within a generalised situation "these kinds of situational problems" (which are typically used as assessment items). I am, therefore, through indexing generalised readers with associated "assumed" expectations, invoking scale. Indeed, my remarks are a good example "scale jumping"Blommaert (2010). The students' linguistic productions (spoken and written) index their own locally developed forms; my intervention indexes language forms and communicational requirements associated with people (such as teachers) and situations (such as assessment) that are socially more widespread and more valued.

Working through this shift in scale takes effort, was quite laborious, and involved several additional scale-jumping invocations, particularly in relation to showing "all their work". Their interactions with me, including the following extract, indicate that this part of their work was quite challenging.

RB: So (.) that's a good beginning (.) but you need to explain like the calculations that you did (.) you need to say what kind of calculations you did.

Curtis: Times

RB: Yup but precisely what did you times what did you add.

Curtis: I timesed seven (.) times seven (.) six times (.)

RB: Right right.

Curtis: Seven plus that's it.

RB: So like when you worked out for purple.

Curtis: I did five times five.

RB: Uhum

Curtis: Plus one.

RB: Right so I would write purple and then exactly what you just said.

The interaction between different speech types is particularly clear in this extract. My use of the word "need", twice, as well as the word "precisely", indexes expected mathematical ways of talking or writing and, indeed, implies they are a requirement. These additional examples of scale-jumping also serve to position the students' formulations as lacking in sufficient scale: By implication, the "good beginning" is constructed as insufficient (both what is good and what is acceptable is determined by RB, of course). In the above extract, moreover, Curtis's account makes use of relatively local forms of mathematical expression, particularly "times", rendered as a participle "timesed". My promptings, however, focus as much on how Curtis formulates his explanation as on its content. Hence, my suggestion that he must explain "the calculations" and then my subsequent drawing out of the details of these calculations index the discourse of mathematics in relation to the construction of an account of a solution. This aspect of mathematical discourse is perhaps more subtle than features like vocabulary. Curtis's eventual written work is shown in Figure 1 (bold type was printed in the problem booklet).

Show all your work:
Purple: I X 5 x 5 = 25
Plus 1 witch is 26
Yellow: I did X 6 x 6 = 36 + 4 = 40
Pink: I did X 7 x 7 = 49 + 9 = 58

The number of tulips needed of each colour is:
26 purple
40 yellow
58 pink

Figure 1. Curtis's eventual written work

Throughout this episode, then, my utterances index "acceptable" ways of talking and writing about mathematics; that is, ways of talking and writing about mathematics that are more widely recognised within a particular mathematical community. The scale-jumping more broadly indexes forms of English language educated discourse, institutional discourses and assessment discourses, through the preference for English and for particular forms of explicit reasoning. This scale-jumping also indexes the students' spoken and written formulations as falling outside of these broader discursive norms, and hence marginalise the students. Through the course of the episode, there is some convergence in the students' utterances towards more conventional mathematical language: For example, in the final version, Curtis includes an account of his choice of operation (multiplication, represented by X), and presents his calculations. Needless to say, there is little reciprocal convergence towards the students' forms of mathematical expression: The printed text cannot be modified and I only used words favoured by the students, such as "times" rather than "multiply" as part of the process of developing more standard mathematical accounts. My interventions often mark aspects of the students' expressions as needing development to be considered more mathematical. When I emphasise the need for an explanation of his calculation, for example, Curtis replies "times". My subsequent prompt "precisely what did you times what did you add" is an example of scale-jumping: It indicates that "times" is not sufficient as an explanation in mathematics and that more specificity is needed for the explanation to fit wider-scale norms of mathematical discourse.

The speech types involved in this episode reflect prevailing orders of indexicality. The students bring speech types from the periphery: Those of Cree-speakers from James Bay for whom English is a second language. Their speech types also include local forms of mathematical language that make sense to the students, either individually or among themselves. The word problem text and I both deploy more authoritative speech types (a form of unitary language), where this authority comes from an indexing of assessment, of the requirements of the genre and of communicating one's work to a generalised other (someone). The heteroglossic encounter between the students, the word problem and me is filled with indexical complexity, but this complexity is ordered; the language of the encounter is stratified, with a hierarchy apparent in which local and peripheral speech types are less valued than more widely standardised forms of mathematical language.

Discussion and Concluding Remarks

Previous research that seeks to examine the socio-political dimensions of language diversity in mathematics classrooms has had a couple of limitations. First, an analysis based on the notion of language-as-a-resource risks focusing attention more on the resources, and less on the socio-political dynamics of interaction. To be sure, such an analysis can be valuable and can highlight inequalities. In the above episode, a focus on resources could examine the resources used by the students, including features of the word problem genre, the diagram, each other's ideas, and so on. This kind of analysis can show how the students are able successfully to interpret the problem, find a solution and write up their thinking. Such demonstrations are important in order to counter deficit perspectives on second language or multilingual learners (see Moschkovich, 2009). Nevertheless, such an analysis can overlook the stratification that arises even in successful encounters with mathematics.

Second, much attention in socio-politically oriented research on language diversity in mathematics classrooms has focused on the specific issue of students' and teachers' choice of language. Planas and Setati-Phakeng (2014) argue that language-as-a-resource is a precursor to language-as-a-right and illustrate their point with examples from situations in which two languages are present and students and teachers take the opportunity to use them. For example, specific words may be translated. In these examples, using more than one language "as a resource" can be seen as part of a possible movement towards the use of students' languages as a right. Language diversity, however, does not always mean that students are able to use more than one language. In Curtis and Ben's class, for example, the teacher does not speak Cree. While the idea that the students should have the right to use Cree in their mathematics class is reasonable, and in fact they often do speak Cree (see Barwell, 2014b), the actual conditions of their classroom place severe limitations on the actualisation of such a right. Put more plainly, code-switching, while tolerated, is a marginal practice in this classroom and those of many bilingual or multilingual students around the world. There is a clear need, therefore, for research to focus on a wider range of issues, beyond that of which language(s) students and teachers should use in mathematics.

The theoretical sociolinguistic ideas I have proposed provide some valuable tools with which to address some of these drawbacks. The concept of heteroglossia highlights the way specific localised utterances in the classroom are linked to broader, stratified, social patterns, whatever language or combination of languages are used. The concept of indexicality facilitates a detailed analysis of this stratification as it plays out in the classroom and makes possible an analysis that links the detail of specific moments of interaction with wider social patterns of marginalisation and stratification. Indeed, it can make visible stratification that occurs even where only one language is used. Finally, the concept of scale-jumping provides a way to identify particular moments in which stratification is produced, such as when teachers explicitly encourage or require students to adopt widely used "standard" forms of language or mathematical discourse.

It is important to understand here that indexicality and stratification are intrinsic features of human interaction. Hence, analyses such as the example with which I have illustrated this paper will not lead to the identification of ways in which to eliminate stratification. Indeed, indexicality is a valuable feature of language, making possible many aspects of interpretation. Blommaert (2010) distinguishes between the indexical order, which refers to the patterns of language that allow us to recognise references to particular groups, activities or situations, and orders of indexicality, which refers to the stratification of language and is implicated in processes of marginalisation. It is through the indexical order that Curtis and Ben recognise a mathematical word problem *as* a mathematical word problem, for example, and are able to work on it and find a solution. Mathematics classroom interaction depends on indexical order to maintain organised forms of mathematical interaction. It is the ordering of indexicalities that is linked to stratification and marginalisation. The ordering of indexicalities means that some forms of language use are socio-politically connected with successful, desirable or powerful people, while others are positioned negatively. In Curtis and Ben's class, much of their repertoire of language practices, such as talking about their mathematical thinking in Cree are not simply under-valued; they are constructed as indexing socially "less good" ways of talking about mathematics. Unfortunately, there is no neat way to decouple these the indexical order from orders of indexicality. Nevertheless, the theoretical ideas I have set out in this paper offer a way forward for research on socio-political dimensions of language diversity in mathematics classrooms. In particular, they provide tools with which to examine the local production of wider social patterns of marginalisation arising in the context of language diversity.

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